ANALYSIS OF TWO-LAYERED GAS-LUBRICATED POROUS BEARINGS

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SUMMARY
A theoretical estimate is made to evaluate performance characteristics in terms of steady state and stability of two-layered externally pressurized porous gas journal bearings. Assuming only radial flow in the porous medium and considering two-dimensional flow in the bearing clearance, basic equations are solved to find load capacity and stability margin. It has been observed that a two-layered bearing gives higher stability when compared with a single-layered one.

Keywords: Porous bearings, externally pressurized, two-layered, steady state and stability

1 INTRODUCTION
Gas-lubricated porous bearings are being used in many industrial applications [1,2]. Because of low frictional characteristics, cleanliness and excellent high temperature properties, gas bearings can be applied in high-speed machinery, e.g. electric motors, ultracentrifuges turbo-machines and high precision instruments, such as missile guidance systems for rockets and spaceships.

Here the compressed gas is fed to the porous bushing at a constant supply pressure. Since the porous medium acts as a bearing surface and a flow restrictor, the supply pressure drops in the porous medium and then in the bearing clearance and exhausts at the atmosphere. The use of two-layered structure would, among its most important advantages, gives the designer the freedom to separate the structural and flow restrictions requirement almost entirely. One could optimize the geometry of the coarse substrate to satisfy the structural requirements, and simultaneously design the fluid flow properties of the thin surface layer with low permeability to suit the designed flow restrictors performance, provided that the pressure drop access the coarse substrate is insignificant (say 5 % or less) compared with that across the restricting layer. By choosing a sufficiently large particle size ratio between the coarse substrate and the thin surface layer, it is possible to control the fluid flow through the two-layered structure such that over 95 % of the pressure drop occurs in the fine pored layer, even when its thickness is only a fraction (e.g. 5 - 10 %) of that of the coarse substrate.

Recent research in Japan [3] and particularly at the Technical University of Munich in Germany [4] suggested that a two-layered bearing can be used for enhancing stability.

Although steady state and stability characteristics for a single-layered bearing are available [5,6], analysis relating to a two-layered one is very limited. With the usual assumptions of porous gas bearings and considering that the flow takes place only in radial direction in the porous bushing, the equation of continuity of flow through porous bushing and the modified Reynolds equation under dynamic condition are written using the first-order perturbation method steady state and dynamic equations in pressure in porous matrix and bearing clearance are obtained. These are then solved using Newton-Raphson iterative method in the porous medium because of its non-linearity. The perturbed equation in bearing clearance come out in linear form and are solved by finite difference method in using successive overrelaxation scheme.

The steady state load carrying capacity is obtained from the steady state pressure. The stiffness and damping coefficients calculated from dynamic pressures are used in the equations of motion to find the stability margin.
2 ANALYSIS

An externally pressurized two-layered porous gas journal bearing is shown in Figure 1. The load is supported by the lubricant film through the pressure generated due to rotation and external pressurization.

The governing equations in porous bushing and the bearing clearance can be written in dimensionless form as:

\[
\frac{\partial^2 \bar{P}}{\partial z^2} - 2\lambda \frac{\partial \bar{P}}{\partial t} = 0
\]

and

\[
\frac{\partial}{\partial \theta} \left( \bar{h}^3 \frac{\partial \bar{P}^2}{\partial \theta} \right) + \left( \frac{D}{L} \right)^2 \frac{\partial}{\partial y} \left( \bar{h}^3 \frac{\partial \bar{P}^2}{\partial y} \right) = 2(1 - 2\lambda \Lambda) \frac{\partial \bar{P}}{\partial z} + 4\Lambda \frac{\partial \bar{P}}{\partial z} + 4 \frac{\partial \bar{P}}{\partial z} = 0
\]

where \( \bar{P}' = \bar{P} / \bar{P}_a \), pressure in the porous matrix

\( \bar{P} = \bar{P} / \bar{P}_a \), pressure in the bearing clearance

\( \bar{h} = \bar{h} / \bar{c} \), film thickness

\( c = \) radial clearance

\( \theta, \bar{y}, \bar{z}, \theta = x/R \),

\( \bar{y} = 2y / L, \bar{z} = z / H \), coordinates

\( \Lambda = \frac{6\eta \omega}{\bar{P}_a \sqrt{\bar{R}}} \), bearing number

\( \omega = \) journal speed

\( R = \) journal radius

\( \gamma = \mu c^2 H^2 / 6kR^3 \), porosity parameter

\( \mu = \) porosity

\( k = \) permeability coefficient

\( H = \) bushing thickness

\( \lambda = \frac{\omega_p}{\omega} \), whirl ratio.

\( \tau = \omega_p t \)

\( \omega_p = \) whirl frequency

\( L = \) length of bearing

\( D = \) diameter of bearing

\( \beta = \frac{12kR^2}{\omega_p c^3 H} \), feeding parameter

It is assumed that at the onset of instability, the position of the journal center can be defined as a steady value \((\varepsilon_0, \phi_0)\) together with a frequency \(\omega_p\), thus

\[\varepsilon = \varepsilon_0 + \varepsilon_1 e^{i\tau}\]

\[\phi = \phi_0 + \phi_1 e^{i\tau}\]

where \(\varepsilon_0\) and \(\phi_0\) are steady state eccentricity ratio and attitude angle and \(|\varepsilon| << |\varepsilon_0|\) and \(|\phi| << |\phi_0|\)

The perturbation equations for pressures and local film thickness can be written as

\[\bar{P}' = \bar{P}'_0 + \varepsilon_1 e^{i\tau} \bar{P}_1 + \varepsilon_0 \phi_1 e^{i\tau} \bar{P}_2\]

\[\bar{P} = \bar{P}_0 + \varepsilon_1 e^{i\tau} \bar{P}_1 + \varepsilon_0 \phi_1 e^{i\tau} \bar{P}_2\]

\[\bar{h} = \bar{h}_0 + \varepsilon e^{i\tau} \cos \theta + \varepsilon_0 \phi_1 e^{i\tau} \sin \theta\]

where \(\bar{h}_0 = 1 + \varepsilon_0 \cos \theta\)

Substituting equation (4) in equations (1) and (2) and neglecting higher powers of \(\varepsilon\) and \(\phi_1\), we get six (three for porous medium and three in the bearing clearance) linear differential equations after separating out \(\varepsilon_0, \varepsilon_1 e^{i\tau}\) and \(\varepsilon_0 \phi_1 e^{i\tau}\).

The steady state equations are:

\[\frac{\partial^2 \bar{P}_0}{\partial z^2} = 0\]

and

\[\frac{\partial^2 \bar{h}_0}{\partial \theta^2} + \frac{3\bar{h}_0}{\bar{h}_0} \frac{\partial \bar{h}_0}{\partial \theta} + \left( \frac{D}{L} \right)^2 \bar{h}_0 \frac{\partial^2 \bar{P}_0}{\partial \theta^2} = 0\]

Similarly, two more sets of equations in \(\bar{P}'_1, \bar{P}'_2\) and \(\bar{P}_2\) can be written following the above procedure.

Solution of equations (5) and (6) gives the steady state pressure distribution which determines load capacity.

The boundary conditions are:

\[\bar{P} = \bar{P}_s \text{ at } \bar{z} = 0\]

\[\bar{P} = \bar{P}_c \text{ at } \bar{z} = \alpha_1\]

\[\bar{P} = \bar{P}_0 \text{ at } \bar{z} = 1\]

\[\frac{\partial \bar{P}_0}{\partial \bar{y}} = 0 \text{ at } \bar{y} = 0\]

\[\bar{P}_0 = 1 \text{ at } \bar{y} = \pm 1\]

and

\[\bar{P}_0(\theta, \bar{y}) = \bar{P}_0(\theta + 2\pi, \bar{y})\]

where \(\alpha_1\) is a fraction of bush thickness and \(\bar{P}_c\) is the pressure at the interface.
Assuming that the pressure drop in the coarse layer is a small percentage of the total pressure drop (say $\alpha_2 = 5\%$) the steady state pressure distribution in the porous bearing can be found after solving equation (5) and satisfying the above boundary conditions. The $P_0$ thus is

$$\frac{-P_0^2}{\bar{P}_0} = \frac{-\bar{P}_s - \alpha_2 (\bar{P}_s - \bar{P}_0)^2}{(1 - \alpha_1)^2} (Z - 1) + P_0^2$$

(8)

and

$$\left[ \frac{\partial \bar{P}_0}{\partial z} \right]_{z=1} = \bar{P}_0^2 - \alpha_2 (\bar{P}_s - \bar{P}_0)^2$$

(9)

Substitution of $\left[ \frac{\partial \bar{P}_0}{\partial z} \right]_{z=1}$ in equation (6), makes the differential equation non-linear in $\bar{P}_0$. This poses a great difficulty in finding an analytical solution. This equation is solved numerically using Newton-Raphson iteration method.

The steady state load is then found from:

$$\bar{W}_s = -\frac{1}{2} \int_{0}^{2\pi} \bar{P}_0 \cos \theta d\theta d\bar{y}$$

and

$$\bar{W}_o = \frac{1}{2} \int_{0}^{2\pi} \bar{P}_0 \sin \theta d\theta d\bar{y}$$

$$\bar{W} = \frac{W}{LDP} = \left( \bar{W}_s^2 + \bar{W}_o^2 \right)^{1/2}$$

(10)

$\phi_0 = \tan^{-1}\left( \frac{\bar{W}_o}{\bar{W}_s} \right)$

This load can be expressed in terms of Sommerfeld number $S = \frac{\eta NLD}{W} \left( \frac{r}{c} \right)^2 = \frac{A}{2\pi W}$, will be inversely proportional to load $\bar{W}$, where $N$ is in revolutions per second.

The dynamics pressures $\bar{P}_1'$, $\bar{P}_1$, $\bar{P}_2'$ and $\bar{P}_2$ are obtained from the linearized differential equations, which are not given here due to space limitation. The stiffness and damping coefficients are found in the following form:

$$\bar{K}_{rr} = -\text{Re} \left[ \frac{1}{2} \int_{0}^{2\pi} \bar{P}_1 \cos \theta d\theta d\bar{y} \right]$$

$$\bar{K}_{oo} = -\text{Re} \left[ \frac{1}{2} \int_{0}^{2\pi} \bar{P}_1 \sin \theta d\theta d\bar{y} \right]$$

(11)

$$\bar{D}_{rr} = -\text{Im} \left[ \frac{1}{2} \int_{0}^{2\pi} \bar{P}_1 \cos \theta d\theta d\bar{y} \right] / \lambda$$

$$\bar{D}_{oo} = -\text{Im} \left[ \frac{1}{2} \int_{0}^{2\pi} \bar{P}_1 \sin \theta d\theta d\bar{y} \right] / \lambda$$

where $\bar{K}_{ij} = K_{ij} c / LDP$ and $\bar{D}_{ij} = D_{ij} c / LDP$.

The other coefficients, $\bar{K}_{11}$, $\bar{K}_{22}$, $\bar{D}_{11}$ and $\bar{D}_{22}$ can be written by analogy.

These are then used in the equations of motion as given by:

$$F_r + W \cos \phi - Mc\omega^2 \left( \dot{\phi} - \epsilon \phi \right) = 0$$

$$F_o + W \sin \phi - Mc\omega^2 \left( \dot{\phi} + 2\epsilon \phi \right) = 0$$

(12)

After non-dimensionalizing equations (12), using equation (11), and for non-trivial solution setting the determinant equal to zero, we get two equations from the real and imaginary parts:

$$\begin{vmatrix}
\frac{\bar{W}}{LDP} & \frac{W}{LDP} & \frac{W}{LDP} \\
\bar{W}_s & \bar{W}_o & \bar{W}_s \\
\bar{W}_o & \bar{W}_s & \bar{W}_o \\
\end{vmatrix} = 0$$

(13)

and

$$\begin{vmatrix}
\bar{K}_{rr} & \bar{K}_{oo} & \bar{K}_{rr} \\
\bar{K}_{oo} & \bar{K}_{rr} & \bar{K}_{oo} \\
\bar{K}_{rr} & \bar{K}_{oo} & \bar{K}_{rr} \\
\end{vmatrix} = 0$$

(14)

where $\bar{M} = \frac{M c^2}{LDP}$ is mass parameter and $M$ is the mass per bearing.

For the calculation of $\bar{M}$ one has to assume a value of $\lambda$ and determine the stiffness and damping coefficients from equation (11) and then the corresponding mass parameter $\bar{M}$ is calculated from equation (13). Then this value of $\bar{M}$ is substituted in equation (14) and seen
whether equation (14) is satisfied. If not, a new value of $\lambda$ is chosen and the process is repeated till it is nearly equal to zero. We have tried to limit the value of this error within -0.001 % to 0.1 %.

3 RESULTS AND DISCUSSION

The steady state and stability characteristics have been compared with single-layered porous bearings. The agreement is good and not reproduced here.

For a bearing working at 5 $\mu$m to 15 $\mu$m bearing clearance, the ratio of permeability coefficient of coarse layer to that of fine layer is around 200. To compare the Sommerfeld number (hence load) and stability parameter of two-layered bearing with that of single-layered bearing, the permeability for the combined effect of the two layers of double-layered porous bearing to be 0.05 time that single-layered bearing with the thickness of fine layer being 0.1 of total thickness. Hence the feeding parameter and porosity parameter increases and decreases respectively by 20 times for the two-layered bearing compared to single-layered bearing.

Figure 2 shows higher load capacity (lower Sommerfeld number) for the two-layered bearing due to reduced seepage into the wall of the ultra-fine inner layer. It has also been found that the stability increased significantly in the case of two-layered bearing (Figure 3).

Like conventional single-layered bearing, it has also been found that higher supply pressure and porosity parameter decreases the stability.

4 CONCLUSIONS

Both steady state and stability of the bearing are improved by using two-layered porous bearing compared to a single-layered porous bearing.

5 REFERENCES