ARC-SHAPED MAGNETIC BEARINGS

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SUMMARY
In many applications, the aim of reducing friction losses can be achieved by using magnetic bearings, either with permanent magnets or electro magnets. Permanent magnets have the important advantage of simplicity but they are intrinsically unstable; on the other side active bearings are very flexible but complicated. The present paper investigates two geometry of arc-shaped magnetic bearings, respectively for radial and axial bearings. The evaluation of forces has been carried out by Finite Element analyses, for three values of the arc opening angle. In addition, a comparison between axially and radially magnetised bearings has been conducted. Results report force versus displacement diagrams that can be useful for evaluating the stiffness of the bearing and for selecting the most suitable element shape for the specific application.

Keywords: permanent magnets, passive magnetic bearings, finite element magnetic analysis.

1 INTRODUCTION
The problem of friction losses and wear damage in bearings, as well as in many other mechanical components, can be conveniently faced by making use of magnetic bearings. Generally the solution resorts to active systems with electro-magnetic elements but they may be rather complicated. On the other hand applications of passive systems are limited because they generate quite low forces and intrinsically unstable, hence requiring a control system. In fact, according to Earnshaw’s theorem [1] a combination of permanent magnets will be unstable at least in one direction, whichever are the shapes and magnetisation of the elements [2-3]. Improvements on the level of forces have been gained with new materials such as rare-earth, that present a residual magnetic induction even greater than 1 Tesla [4-6].

The question of the intrinsic instability cannot be overcome. Thus in previous works [7-8], the study of the element shapes and magnetisation has been faced in order to reduce unstable forces, with particular attention to the conic shape. With the same aim, the present paper investigates the behaviour of arc-shaped magnetic bearings.

2 BEARING GEOMETRY
The arc-shaped bearing is considered to be made up of two slices of spherical shells. Two different geometry have been assumed, one for the axial bearing and one for the radial bearing.

The axial bearing geometry is shown in Fig.1 in a concentric position; the axis of symmetry corresponds to the shaft axis and also to the load direction. The centre of the spheres is the point O and the angle \( \phi \) defines the opening angle of the bearing elements. The axial bearing elements are limited by a plane normal to the shaft axis which passes through the centre of the sphere. Reflecting the elements with respect to such plane, the radial bearing shown in Fig.2 can be obtained.

It is worth noting that in this case the angle \( \phi \) is the half-opening angle of the shell.

Fig. 1: Axial bearing (section and half-bearing).

A reference geometry has been defined for Finite Elements calculations; for both types of bearing the following dimensions have been assumed: inner shell radius \( r_{i} = 15 \text{ mm} \), outer shell radius \( r_{o} = 35 \text{ mm} \) and thickness of the shells \( t = 15 \text{ mm} \) (see Fig.2 for notation).

Fig. 2: Radial bearing (section).

Three values of the angle \( \phi \) have been considered: 15°, 30° and 45°.

Residual magnetic induction is assumed to be 1 T, as for rare earth materials and the magnetisation is supposed to have both axial and radial direction.

3 EVALUATION OF FORCES
The force that acts between the elements of the bearing is evaluated for different relative positions. The outer element is supposed to be fixed while the inner element translates in the axial and in the radial direction.
Forces have been evaluated by Finite Elements analyses, using Ansys® program, Emag package. Two different models have been used respectively for 2D and 3D analyses. The first model, based on magnetic potential vector approach, is used for axial-symmetric configuration of the bearing, when only the axial translation \( z \) is considered (Fig. 3). The 3D mesh introduces a radial displacement \( e \) and each node has one degree of freedom: the magnetic scalar potential.

It is evident from eq.(2) that almost one direction is unstable. When the bearing elements have coincident axis, the problem is axial symmetric and eq.(2) may be expressed in cylindrical co-ordinates as follows:

\[
\frac{\partial F_z}{\partial z} + 2 \frac{\partial F_r}{\partial r} > 0
\]

so the definition of radial and axial bearing referred to the stability should be clear.

In the following, forces that act on the inner element are reported in dimensionless form. They are scaled by a factor \( f \) which depends on the magnetisation of both elements, on magnetic free space permeability and on a geometric parameter \( A_m \):

\[
f = \left( \frac{\mu_0 M_i M_o A_m}{4\pi} \right)
\]

\( A_m \) is assumed to be the area of the mean shell with respect to the bearing elements.

### 4 RESULTS

For the reference geometry defined above, the following results have been obtained. At first, forces for the axial-symmetric configuration (\( e = 0 \)) are reported.

\[
\frac{\partial F}{\partial x_i} < 0 \quad i=1..3
\]

The force field created by permanent magnets, that have a relative magnetic permeability greater than 1, satisfies also the condition of positive divergence:

\[
\frac{\partial F_i}{\partial x_1} + \frac{\partial F_j}{\partial x_2} + \frac{\partial F_k}{\partial x_3} > 0
\]

In Fig.5 c) e d) results relative to the axial bearing are shown, for three angles (15°, 30° and 45°) and for radial and axial magnetisation, described respectively in Fig.5 a)-b).
Results report the dimensionless axial force $F'_z = F_z/f$ versus the dimensionless axial displacement $z' = z/(re + t)$. In both cases, forces are negative as they tend to join the elements. In the bearing with radial magnetisation (Fig.5.c), for small arc opening angles the force grows markedly with $z'$, while for greater values of $\phi$ the rate appears reduced. In case of axial magnetisation – except for $\phi = 15^\circ$ –, forces (Fig.5 d) are higher but almost constant as $z'$ varies, that means a very low stiffness. This may be an important disadvantage since the position of shaft changes markedly for small variations of the external force. For the global equilibrium, also an external force must be assumed to act on the shaft, which is carried by the inner element of the bearing.

It is worth noting that, in some cases, for displacements greater than about 0.1, the bearing becomes unstable in the axial direction.

The difference in the trends between the different kind of magnetisation, may be explained by analysing the magnetic flux lines, as shown in Fig.6. In the axially magnetised bearing opposite poles are adjacent and lines pass directly from one element to the other. On the other hand, with radial magnetisation many lines close within the same element with a reduced interaction of field.

![Fig. 6: Axial bearing: magnetic flux lines.](image)

In an analogous way, results for the radial bearing are reported in Fig.6, for $\epsilon = 0$. Also in this case, the axial force has been evaluated versus the axial displacement and from it the radial stiffness may be obtained, according to eq.(3).

Due to the symmetry of the position, when $z = 0$ both radial and axial forces are zero. In case of radial magnetisation (observe that inner and outer element have opposite magnetisation direction) the axial force grows with positive rate. As a consequence, according to eq.(3) the derivative of the radial force with respect to the radial displacement –the radial stiffness-, is positive so the bearing is stable in the radial direction. The behaviour of the bearing can be approximately linearized in the neighbourhood of the origin and also the relationship between the radial force and the radial displacement can be obtained. It is worth noting that when the angle $\phi$ is equal to $45^\circ$, the force is almost zero. In fact as a limit, two spherical shell magnets one inside the other, radially magnetised do not generate forces.

When the magnetisation is axially directed and equiviruse in the elements, the kind of the bearing may change from radial to axial with the angle $\phi$.

![Fig. 7: Radial bearing: magnetisation directions (a-b) and correspondent dimensionless axial force (c-d).](image)

In particular, Fig.7d) shows that $\phi = 15^\circ$ corresponds to a radial bearing while $\phi = 45^\circ$ to an axial one.

Even in this case further increases in $\phi$ will reduce to zero the force. Also for the radial bearing the lines of magnetic flux are reported below in Fig.8.

![Fig. 8: Radial bearing: magnetic flux lines.](image)

Previous results may be used to analyse the complete behaviour of the bearing. As already explained, according to eq. (3), approximated curves can be obtained for small values of $\epsilon$. In fact the axial symmetric analyses permit to evaluate the derivative of the axial force with respect to axial displacement and from it the correspondent radial derivative is soon determined.
In Fig. 9 an example of dimensionless radial force versus dimensionless radial displacement \((e' = e/(re + t))\) is represented, relative to the radial bearing geometry with \(\phi = 15^\circ\) and axial magnetisation. Both linear approximations for small values of \(e'\) and Finite Element results are reported.

![Fig. 9: Comparison between 3D-Finite Element (FE) and approximated (Appr) results.](image)

It can be observed that for \(e' < 0.03\) approximated results are in very good agreement with FE data. When small displacements from coaxial position are expected, plane analyses are sufficient to have information about the behaviour of the bearing. An analogous diagram is reported in Fig.10 for the radial bearing, still with \(\phi = 15^\circ\) and axial magnetisation. Even in this case, the linear approximation is adequate for describing the system behaviour when \(e' < 0.025\).

![Fig.10: Radial bearing: comparison between FE and Appr results (see Fig.9 for legend).](image)

It must be observed that values of \(e'\) and \(z'\) are limited by the condition of contact between the elements.

In Fig.11 the axial force corresponding to for the same case of Fig.9 is reported for different radial displacement.

![Fig. 11: Axial bearing: axial force for different e’.](image)

5 CONCLUSIONS

Arc-shaped magnetic bearings have been investigated, with particular reference to two geometry one for the radial and one for the axial bearing. Forces have been evaluated by means of FE analyses with an accuracy of \(5 - 10\%\). Results compare axial and radial magnetisation that offer different advantages in terms of stiffness and value of forces. An approximated linearisation of the behaviour of the bearing for small values of the eccentricity is also discussed and can be useful for avoid 3D-analyses that require a very long time.

The study may help the selection of the most suitable bearing, in terms of shape and magnetisation direction, for the specific application. In particular this kind of bearings may be advantageous in micro-machines.

6 REFERENCES


