STUDY OF THE ROLLING CONTACT FATIGUE IN LUBRICATED CONTACTS

G. DONZELLA, A. MAZZÙ and L. SOLAZZI
Department of Mechanical Engineering, University of Brescia, via Branze 38, 25123 Brescia, ITALIA; e-mail: donzella@bsing.ing.unibs.it, mazzuan@bsing.ing.unibs.it, solazzi@bsing.ing.unibs.it

SUMMARY
In components subjected to rolling contact fatigue the most dangerous failure mechanisms, such as macropitting and subcase fatigue, are related to mode II propagation of subsurface cracks, as some experimental results show. Thus, a fracture mechanics – based study of these phenomena involves both the calculation of applied mode II SIF at the crack tip, and an estimation of the threshold SIF. In this paper a method for calculating applied SIF under an arbitrary load distribution and its application to problems of elastohydrodynamic lubrication are presented. Such method is based on a mode II weight function for subsurface cracks, which depends just on the geometry of the system. The resulting SIF can be then compared to the threshold SIF of short – cracks, which simulate the presence of inherent defects. In this way, a general criterion for predicting crack propagation in contact problems is formulated, and it is shown that in most cases of subsurface fatigue problems the lubricated contacts can be treated as hertzian contact problems.

Keywords: Rolling contact fatigue, Elastohydrodynamic lubrication, Subsurface cracks, Mode II propagation

1 INTRODUCTION
Different fatigue phenomena can occur in components subjected to rolling contact; in particular, they can involve the near – surface region, (micropitting), or subsurface regions, as it happens when macropitting or subcase fatigue occur. The fatigue phenomena occurring at higher depths are the most dangerous, because they are usually sudden and cause heavy damages to the components involved. The experimental evidence [1] is that in this kind of failure the leading mechanism is mode II propagation of subsurface cracks. Especially in hard materials, such cracks often emanate from inclusions or defects and propagate along a direction parallel to the rolling speed, until they branch towards the surface. These phenomena can start at the depth where the applied shear stress are the highest, or, in surface hardened components, near the interface between the hardened case and the soft core [2].

An approach due to Murakami [3,4] shows that inclusions or voids or other defects, which are intrinsically present in the real components, can be treated as micro-cracks; as a consequence, the crack nucleation problem can be neglected. Thus, according to the fracture mechanics principles, a criterion to predict fatigue failure in such components has to take into account both the applied variation of mode II SIF in a load cycle (\( \Delta K_{II} \)) and the corresponding threshold value (\( \Delta K_{IIth} \)). If the applied \( \Delta K_{II} \) overcomes the \( \Delta K_{IIth} \), the crack propagates. The fatigue limit of the material has to be intended as the load level at which the cracks do not propagate.

The determination of \( \Delta K_{II} \) and \( \Delta K_{IIth} \) however, is not simple.

The \( \Delta K_{IIth} \) for the so – called long cracks is a property of the material; but it’s not clear if this statement is valid also for cracks whose length is comparable to the intrinsic defects dimensions (short cracks). It was shown by Murakami et al. [4] that for short cracks propagating in mode I the \( \Delta K_{Ith} \) depends both on material hardness and crack length. As concerning the \( \Delta K_{IIth} \) for short cracks, Donzella et al. [5] have proposed a model which considers two bounds for the \( \Delta K_{IIth} \): the upper bound descend from the hypothesis that in mode II there is not any short crack effect, i.e. \( \Delta K_{IIth} \) is a property of the material; the lower bound derives from the hypothesis that \( \Delta K_{IIth} \) is a function of hardness and crack length according to a law analogous to the one valid for the \( \Delta K_{Ith} \). Some experimental results in rolling contact fatigue [6] seem to be closer to the lower bound, which is expressed by the following:

\[
\Delta K_{IIth} = 2.5 \times 10^{-1} \cdot (HV + 120) \left( \frac{1}{A} \right)^{1/2}
\]

The applied \( \Delta K_{II} \) depends on the system geometry, crack length and stress distribution. In the literature several results regarding Hertz contact problems are present [7,8]; in particular, Donzella et al. [5] found a simple algebraic function for calculating \( \Delta K_{II} \) under Hertzian conditions for short cracks parallel to the surface at different depths. Anyway, there is no method applicable to non – hertzian contacts, such as lubricated contacts. In this paper a method for calculating the \( \Delta K_{II} \) for a two – dimensional subsurface crack under an arbitrary load distribution is presented. It is based on a weight function [9] which depends just on the crack depth.

Its application to load distributions due to elastohydrodynamic lubrication allows to evaluate the rheological parameters effect on the applied SIFs and the study of fatigue phenomena in lubricated contacts between cylindrical bodies.

2 NUMERICAL PROCEDURE
The lubricated contact between cracked cylindrical bodies is simulated by the two – dimensional plane strain model reported in figure 1. The lubrication is supposed to be elastohydrodynamic.
The first step of the calculation procedure is the determination of the contact actions following the numerical method described in [10]. The distribution of pressure is calculated by the Newton–Raphson iterative method as proposed by Houpert and Hamrock [11]; then, the distribution of the contact shear stress is derived by the Eyring model [12].

Finally, the weight function (WF) method can be applied to calculate the mode SIF at the crack tip. For an internal crack propagating in mode II, the WF is a function $h_{II}(x, a)$ such that:

$$K_{II} = \int_{0}^{a} h_{II}(x, a) \tau(x) dx$$  \hspace{1cm} (2)

where $A$ is the index identifying the left crack tip, $a$ is the crack half–length and $\tau(x)$ is the shear stress distribution along the crack front in the uncracked body. The WF for a crack in a two–dimensional half space was calculated in the following form:

$$h_{II}(x, a) = \begin{cases} \frac{2}{\pi a} \sum_{m=1}^{\infty} D_{m} \left(1 - \frac{x}{a}\right)^{m-1/2} & \text{for } 0 < x < a \\ \frac{2}{\pi a} \sum_{m=1}^{\infty} \tilde{D}_{m} \left(1 + \frac{x}{a}\right)^{m-1/2} & \text{for } -a < x < 0 \end{cases}$$  \hspace{1cm} (3)

where the coefficient $D_{m}$ and $\tilde{D}_{m}$ depend on the depth $z$. They were derived from several cases of Hertz contact whose $K_{II}$ had been calculated by FEM models; anyway, this WF has a general validity, and applies also to stress fields due to non–Hertzian contacts.

### 3 RESULTS AND DISCUSSION

Several analyses were made in order to study the effects of the working conditions on $K_{II}$. The variables examined and their range of values are reported in table 1. The values of nominal viscosity and Roelands parameter were chosen in order to simulate the characteristics of the most common lubricants [14]. The values of Hertz contact pressures and half widths cover the range of many applications, from cams to railway wheels. The variation of the Eyring stress, which is a parameter used to define the contact shear stress curve, was neglected, because its effect on the stress field is very weak, as shown in [10]; the constant value of 10 MPa was assigned to this parameter. The crack lengths were chosen in the range of the short – cracks, while the depths considered correspond to a range containing the highest shear stresses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_0$ [MPa]</td>
<td>Nominal Hertz pressure</td>
<td>500 – 1500</td>
</tr>
<tr>
<td>$b$ [mm]</td>
<td>Nominal Hertz contact width</td>
<td>0.1 – 6</td>
</tr>
<tr>
<td>$\eta_0$ [Pa s]</td>
<td>Lubricant viscosity at atmospheric pressure</td>
<td>0.005 – 0.15</td>
</tr>
<tr>
<td>$Z$</td>
<td>Roelands parameter (used to express $P - \eta$ dependence)</td>
<td>0.5 – 0.67</td>
</tr>
<tr>
<td>$U$ [m/s]</td>
<td>Mean rolling speed</td>
<td>0.1 – 3</td>
</tr>
<tr>
<td>$U/U$</td>
<td>Sliding ratio = sliding speed – mean speed ratio</td>
<td>0 – 0.04</td>
</tr>
<tr>
<td>$a$ [µm]</td>
<td>Crack length</td>
<td>20 – 40</td>
</tr>
<tr>
<td>$z$ [mm]</td>
<td>Crack depth</td>
<td>0.08 – 3.6</td>
</tr>
</tbody>
</table>

Table 1: Parameters examined in the model

As was shown in [10], the difference between the stress field due to a lubricated contact and the one due to a Hertzian contact is mainly caused by the fact that in the former a contact tangential traction is present which is neglected by the Hertz model; differences in the pressure distributions, instead, have not a significant effect. Again in [10], the most strongly influencing parameters are indicated as the nominal viscosity $\eta_0$, the Roelands parameter $Z$ and the nominal Hertz pressure $P_0$, as they significantly influence the contact traction curves. Starting from this indication, the effects of these parameters on the $K_{II}$ curves were investigated at first.

In figure 2 we can observe the effect of $\eta_0$ on the variation of $K_{II}$ during a pass of the load above a crack situated at $z = 0.4 \, b$. The mode II SIF is also plotted in the case of a Hertz contact with a traction curve obtained multiplying the punctual values of pressure by the friction coefficient of 0.2, which is much higher than the values typical of the elastohydrodynamic lubricated contacts. While the maximum value of $K_{II}$ varies with different lubricating and friction conditions, $\Delta K_{II}$ tends to be almost constant: the curve of $K_{II}$, indeed, seems to translate on the diagram maintaining the same aspect. The same behaviour is evident in figure 3 where the maximum $K_{II}$ and $\Delta K_{II}$ are plotted in the dimensionless form against the depth. Whilst the $K_{II}$ curves are clearly distinct, the $\Delta K_{II}$ curves tend to overlap. Analogous effects were observed with variations of $Z$, $P_0$, and the other rheological parameters. Thus, it can be affirmed that in the range of values considered the tangential actions at the contact surface do not affect the propagation of this kind of cracks, and the lubricated contacts, with respect to the fatigue effects, can be approximated by Hertz contacts. This is true for a friction coefficient not exceeding 0.3, because otherwise the maximum applied $\tau_{xz}$, on which mainly depends $\Delta K_{II}$, is located at the surface, and not in the subsurface region. Anyway, such friction coefficient cannot be reached in lubricated contacts.
In [5] the following algebraic function for calculating length and geometrical parameters (i.e. depth Hertzian parameters (i.e. pressure values: $c_0 = 0.21632$, $c_1 = 1.49316$, $c_2 = -2.7232$, with the constant coefficients having the following limited to the field where contact surface proximity is negligible; as highest shear stresses are located at as approaches zero, the (6) tends to the (4). The maximum error which can be made using the (5) and the (6) with respect to the numerical methods is about 10%.

4 APPLICATION TO FATIGUE LIMIT PREDICTION

The (1), the (5) and the (6) can be used to predict the fatigue limit in components subjected to cyclic contact. As in [5], a failure risk index $I$ can be defined:

$$I = \frac{\Delta K_f(P_0, a/z, a/b)}{\Delta K_{f,0}(HV(z),a)}$$

where $HV$ is the Vickers hardness of the material, which is expressed as a function of the depth $z$ in the case of surface hardened components. With respect to the form proposed in [5] there are two differences: the first, obviously, is that in the (7) $I$ depends also on the ratios $a/b$ and $a/z$; the second is that just the lower bound for $\Delta K_{f,0}$ is considered, because it is assumed that a short crack effect is present in mode II propagation. Curves similar to those represented in figure 4 can so be easily plotted, and an evaluation of the failure risk and of the critical depth can be made, although at the present it must be considered indicative, because the elaboration
of the correct model for $\Delta K_{th}$ determination is actually in progress.

![Graph](image)

**Figure 4:** plots of the applied SIF, threshold SIF and failure index in a surface hardened component with $P_0=2000$ MPa, $b=1$ mm, $2a=36 \mu m$

## 5 CONCLUSIONS

A method based on a weight function for calculating the mode II SIF for subsurface cracks in a half - space was presented. It is applicable to components subjected to contact loading, also in the case of non – hertzian conditions.

By this method the influence of the lubrication conditions was studied. It was shown that, in the range of values considered, the rheological parameters and, in particular, the frictional effects do not affect the $K_{II}$ range during a load cycle, but just its maximum. As the fatigue behaviour depend on the amplitude $\Delta K_{II}$, it can be concluded that for subsurface cracks the lubrication and friction conditions have not significant influence on propagation, and the loading condition are well approximated by the Hertz model.

An algebraic formula was obtained for calculating the applied $\Delta K_{II}$ as a function of geometrical parameters (crack depth and crack length) and hertzian parameters (nominal pressure and contact half – width). Such formula, coupled to a model for the short crack threshold SIF $\Delta K_{th}$ published by the Authors, allows a fast evaluation of the failure risk in components subjected to contact loading, also considering the case that they have been surface hardened.

## 6 REFERENCES