IDENTIFICATION OF THE OPERATING PARAMETERS OF TILTING PAD JOURNAL BEARINGS

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SUMMARY
The paper describes a procedure to identify the operating parameters of a mechanical system comprising a rigid rotor on two tilting pad journal bearings. The dynamics of this system is strongly influenced by the geometrical characteristics of the bearings that aren’t easily measurable, and can also vary during the operating conditions. The illustrated procedure is based on the Extended Kalman Filter (EKF). Starting from the experimental signals relative to the rotor axis motion, the EKF identification technique allows estimating the values of the parameters of a mathematical model that describes the dynamic behaviour of the rotor bearings system. The experimental tests has been conducted on an apparatus that permits to modify some operating parameters, as the geometrical preload, the pad offset, the load configuration, the rotor unbalance and the rotor mass, in order to obtain a wide range of operational conditions. The rig allows also determining the dynamic coefficients by means of the rotating force method.

Keywords: tilting-pad journal bearings, lubrication, rotor-bearing system, parametric identification.

1 INTRODUCTION
The tilting pad journal bearings are often used in turbo-machinery, as they are very stable at high speeds. This quality is determined by the complexity of such bearings especially if compared with the traditional sleeve journal bearings. In fact (fig. 1) the combination of the main geometrical parameters, as the pad offset, the geometrical preload, and the configuration of the external load with respect to the bearings’ pads, gives to the bearing such stiffness and damping characteristics that make the system stable in a wide range of operating conditions.

Nevertheless, during the operation, the value of the above geometrical parameters can vary, due for example to thermal deformations or to wear, and the rotor bearings system could exhibit great amplitude dangerous vibrations or instability phenomena [1]. For this reasons, during the operation, the actual values of all these parameters should be evaluated in order to prevent malfunctions. The identification techniques represent a valid instrument to conduct the above evaluation during the running, and also they can result useful in order to apply control strategies.

The parametric identification techniques consist in going back to the unknown values of some parameters of a mathematical model, which describes the behaviour of a real system, starting from the knowledge of a set of experimental measurements relative to the system itself.

Fig. 1  Tilting-pad journal bearing scheme
Generally (fig.2) it is necessary: (i) to force both the real system and the mathematical model with a known input \( u(t) \), (ii) to acquire the time history of some output from the real system, and finally, acting on the value of the model’s parameters (iii) to make equal its output to that of the real system.

![Real system and model](image)

Fig.2 Identification procedure

For a real system consisting of a rigid, symmetrical rotor on two five pads tilting pad journal bearings, it is possible to individuate two different approaches depending on the type of investigation that have to be made:

a) with reference to the stability analysis of the steady state equilibrium position, it is possible to describe the rotor axis motion in the neighbourhood of this position by means of a relatively simple two d.o.f. linear model, characterised by the presence of eight parameters, i.e. the equivalent stiffness and damping coefficients. In this case the estimation of the parameters’ values can be conducted either by analytical or numerical deterministic techniques [2, 3];

b) if the estimation of the values of some bearings’ characteristic parameters (i.e. geometrical preload, radial clearance, etc.) is required, then a more complex non-linear model, characterised by the presence of a great number of parameters, must be adopted. In such a case the identification technique could consist in a statistical method as the Extended Kalman Filter (EKF).

In the following a procedure based on the EKF is illustrated, together with some results of an experimentation conducted on a test rig.

2 A METHOD BASED ON THE EXTENDED KALMAN FILTER

2.1 The theoretical model

If the rotor-bearing system is symmetrical to the centre plane orthogonal to the rotor axis, it can be studied referring to a system comprising a single bearing and a journal with mass \( M \) equal to half the rotor mass. A set of \( 2 + n \) dimensionless non-linear equations of motion can be written, where \( n \) is the number of bearing pads [1]:

\[
M \ddot{x} = f_x(x, y, \dot{x}, \dot{y}, A_j, \dot{A}_j) + M \rho \cos \tau
\]

\[
M \ddot{y} = f_y(x, y, \dot{x}, \dot{y}, A_j, \dot{A}_j) - \frac{1}{\sigma} + M \rho \sin \tau
\]

\[
I \ddot{\theta}_j = T_j(x, y, \dot{x}, \dot{y}, A_j, \dot{A}_j)
\]

The hydrodynamic force components \( f_x \) and \( f_y \) and the moments \( T_j \) of the pressure forces with respect to the pad pivots are obtained by integrating the Reynolds lubrication equation, under the hypothesis of laminar and isothermal flow which, in the dimensionless form, can be written as follows:

\[
\frac{\partial}{\partial \theta} \left( h^3 \frac{\partial p}{\partial \theta} \right) + \left( \frac{R}{L} \right)^2 \frac{\partial^2}{\partial z^2} \left( h^3 \frac{\partial p}{\partial z} \right) = 6 \frac{\partial h}{\partial \theta} + 12 \frac{\partial h}{\partial \tau}
\]

Assuming for the pressure a parabolic pattern in the axial direction:

\[
p(\theta, z) = p(\theta, 0) \left[1 - (2z)^2\right] - \frac{1}{2} \leq z \leq 1/2
\]

equation (2) can be written in the following one-dimensional form:

\[
\frac{d}{d\theta} \left( h^3 \frac{dp(\theta,0)}{d\theta} \right) - \frac{2}{L} \left( \frac{D}{L} \right)^2 h^5 p(0,0) = 6 \frac{dh}{d\theta} + 12 \frac{dh}{d\tau}
\]

In the Reynolds equation the oil film thickness \( h(\theta) \) assumes the following dimensionless expression:

\[
h(\theta) = 1 - x \cos \theta - y \sin \theta + A_j \sin(\theta_{pj} - \theta) - m \cos(\theta_{pj} - \theta) + h_{sp}
\]

The film thickness for each pad depends on the journal position, the inclination \( A_j \) of the \( j \)-th pad, the geometrical preload \( m \), and the geometry of the leading edge of the pad, of height \( d_{sp} \) and angular extension \( \theta_{sp} \) (Fig. 3), through the term \( h_{sp} \), commonly called spragg relief, for which the following expression is assumed:

![Fig. 3 Spragg relief](image)

\[
\overline{h}_{sp} = \begin{cases} 
\frac{d_{sp} - (d_{sp}/R \theta_{sp}) \theta}{\theta_{sp}} & \text{for } 0 \leq \theta \leq \theta_{sp} \\
0 & \text{for } \theta \geq \theta_{sp}
\end{cases}
\]

The theoretical model does not take into consideration the elastic deformation of the pads.

In conclusion, under the hypothesis of five equal pads, the above model consists in a 14 state variables system with 14 parameters, as summarised in the following table.

<table>
<thead>
<tr>
<th>State variables</th>
<th>for</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x, y, \dot{x}, \dot{y}, A_j, \dot{A}_j )</td>
<td>( j = 1,5 )</td>
<td>( M, I, \rho, \sigma, R_{sp}, R_{b}, L, \theta_{sp}, \beta, m, d_{sp}, x_0, y_0, \theta_{sp} )</td>
</tr>
</tbody>
</table>
Of course, a mathematical model accounting for the pads to be different one from the others, would contain a greater number of parameters.

2.2 The EKF identification technique

The identification technique adopted here, namely the continuous Extended Kalman Filter, calls for the model and the measurements to be available in the form of continuous functions of time. It is thus necessary to have measurements sampled at a sufficiently high frequency so that they can be made continuous during algorithm execution through linear interpolation.

Under this hypothesis, the dynamic model and the measurements can be put in the following form:

\[ \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t) + \mathbf{w}(t) ; \quad \mathbf{w}(t) = \mathbf{N}(\mathbf{0}, \mathbf{Q}(t)) \]

\[ \mathbf{z}(t) = \mathbf{h}(\mathbf{x}(t), t) + \mathbf{v}(t) ; \quad \mathbf{v}(t) = \mathbf{N}(\mathbf{0}, \mathbf{R}(t)) \]

The vectors \( \mathbf{x} \) and \( \mathbf{f} \) have the following structure:

\[
\mathbf{x}(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_{14}(t) \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_p \end{bmatrix} \quad ; \quad \mathbf{f}(t) = \begin{bmatrix} f_1(\mathbf{x}(t), t) \\ f_2(\mathbf{x}(t), t) \\ \vdots \\ f_{14}(\mathbf{x}(t), t) \end{bmatrix} \]

in which \( s_j(t) \) indicates the 14 states and \( p_j \) indicates the parameters in the above table which have to be identified. Consequently, the first 14 terms in the vector \( \mathbf{f}(t) \) are deduced from the motion equation (1), while the final part of the vector contains some 0's to impose the fact that the parameters do not vary over time. The overall size \( n \) of the two vectors is thus given by the sum of the 14 system states and the \( p \) parameters to be identified.

The vector \( \mathbf{z}(t) \) of the measurements has a size \( m \) equal to the number of output signals recorded during a test and is, in the most general case, dependent on the system state through the non-linear relation \( \mathbf{h}(\mathbf{x}(t), t) \).

Finally the model noise \( \mathbf{w}(t) \) and the measurement noise \( \mathbf{v}(t) \) are assumed to have a null mean and have variance matrices \( \mathbf{Q}(t) \) and \( \mathbf{R}(t) \) respectively.

Having posed the problem in the form described above, the value of the \( p \) parameters \( p \), together with the estimate of the 14 states \( s \), is provided by the steady state solution of the following system of first order differential equations [4 to 9]:

\[ \dot{\hat{\mathbf{x}}}(t) = \mathbf{f}(\hat{\mathbf{x}}(t), t) + \mathbf{K}(t) \left[ \mathbf{z}(t) - \mathbf{h}(\hat{\mathbf{x}}(t), t) \right] \]

\[ \mathbf{P}(t) = \mathbf{F}(\hat{\mathbf{x}}(t), t)\mathbf{P}(t) + \mathbf{P}(t) \mathbf{F}^T(\hat{\mathbf{x}}(t), t) \]

\[ \mathbf{Q}(t) - \mathbf{P}(t) \mathbf{H}^T(\hat{\mathbf{x}}(t), t) \mathbf{R}^{-1}(t) \mathbf{H}(\hat{\mathbf{x}}(t), t) \mathbf{P}(t) \]

\[ \mathbf{K}(t) = \mathbf{P}(t) \mathbf{H}^T(\hat{\mathbf{x}}(t), t) \mathbf{R}^{-1}(t) \]

where the matrices \( \mathbf{F} \) and \( \mathbf{H} \) are given by:

\[ \mathbf{F}(\hat{\mathbf{x}}(t), t) = \left[ \frac{\partial \mathbf{f}(\mathbf{x}(t), t)}{\partial \mathbf{x}(t)} \right] \quad \mathbf{h}(t) = \hat{\mathbf{x}}(t) \]  

\[ \mathbf{H}(\hat{\mathbf{x}}(t), t) = \left[ \frac{\partial \mathbf{h}(\mathbf{x}(t), t)}{\partial \mathbf{x}(t)} \right] \quad \mathbf{h}(t) = \hat{\mathbf{x}}(t) \]

Considering that the estimate error variance matrix \( \mathbf{P} \) is symmetrical and that, therefore, only the terms of the main diagonal and one of the two triangles need to be calculated, the number of differential equations in the system (5) is given by:

\[ N = n + (n \cdot n - n) / 2 + n = (n^2 + 3n) / 2 \]

In the present paper the solution of the system of \( N \) differential equations (5) was obtained by adopting the Adams-Moulton method, starting from known initial conditions for the state and approximated conditions for the parameters and the terms of the matrix \( \mathbf{P} \). At each step of the integration, the terms of the \( \mathbf{F} \) and \( \mathbf{H} \) matrices need to be updated by numerically executing the derivatives in (6).

3 THE TEST RIG

The test rig, set up at the DIME laboratory (fig. 4), is equipped with bearings, supplied by Federal Mogul®, that allows to vary the geometrical preload, by inserting calibrated shims (0.012mm thick) between the pad body and the pivot (fig.5). Two pairs of bearings have been adopted for the tests respectively characterized by the offset \( \beta_1/\beta_2 \) values of 0.5 and 0.6, being equal all the other dimensions [10].

![Fig. 4 The test rig](image)

![Fig. 5 Bearing pad](image)

In the center of the rotor shaft (fig. 4) is mounted a disk which can be left idle with respect to the shaft. This disk can be unbalanced and made to rotate, by means of jets of compressed air, at a speed that may be different from that of the rotor to create a rotating load acting on the
rotor, which can allow to identify the dynamic coefficients [2,3].

The apparatus is instrumented to acquire the rotor and the disk speeds, the oil temperature in the bearings and the time history of the shaft rotor axis motion.

The main nominal apparatus characteristics are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor mass</td>
<td>$M = 28\text{kg}$</td>
</tr>
<tr>
<td>Journal radius</td>
<td>$R = 19.9735\text{mm}$</td>
</tr>
<tr>
<td>Radial pad clearance</td>
<td>$C_p = 0.062\text{mm}$</td>
</tr>
<tr>
<td>Radial bearing clearance</td>
<td>$C_b = 0.0325\text{mm}$</td>
</tr>
<tr>
<td>Distance between bearings</td>
<td>$L_b = 0.20\text{m}$</td>
</tr>
<tr>
<td>Bearing length</td>
<td>$L = 16 \text{mm}$</td>
</tr>
<tr>
<td>Bearing radius</td>
<td>$R_b = 20.006\text{mm}$</td>
</tr>
<tr>
<td>Pad radius</td>
<td>$R_p = 20.0355\text{mm}$</td>
</tr>
<tr>
<td>Length/Diameter</td>
<td>$L/D = 0.4$</td>
</tr>
<tr>
<td>Pivot offset</td>
<td>$\beta_p/\beta = 0.5: 0.6$</td>
</tr>
<tr>
<td>Geometrical preload</td>
<td>$m = 0.06 - 0.58$</td>
</tr>
<tr>
<td>Angular amplitude of pads</td>
<td>$\beta = 60^\circ$</td>
</tr>
</tbody>
</table>

### 4 AN APPLICATION OF THE EKF IDENTIFICATION TECHNIQUE

To test the goodness of the EKF identification procedure a first application has been developed in order to identify the following geometrical parameters: the actual coordinates, $x_o, y_o$, of the bearing centre with respect to the displacement transducers and the geometrical preload, $m$, of the five pads.

The need to identify the centre position derive from the fact that the motion equations (1) are referred to a fixed frame with origin in the bearing centre, i.e. the centre of the circle of radius $R_b$ tangent to the pads, whose experimental location is particularly difficult.

Another of the most difficult quantities to determine is the actual value of the geometrical preload:

$$m = 1 - (R_b - R / R_p - R) = 1 - C_b / C_p$$

Because it depends on three quantities, each one of them afflicted with dimensional tolerances.

For this reason for a real plant, starting from the design dimensions and the relative tolerances, it’s only possible to determine a wide range of preload values, rather than the actual value.

As an example, for the bearings adopted, the above design quantities result:

- Journal radius $R$: 19.9655 + 19.9735 mm;
- Bearing radius $R_b$: 20.0 ÷ 20.03 mm;
- Pad radius $R_p$: 20.0355 ÷ 20.0435 mm.

Consequently, considering all the possible combinations, it results:

Bearing without shims:

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$R_p$</td>
<td>20.0355</td>
<td>20.0355</td>
<td>20.0355</td>
<td>20.0435</td>
</tr>
<tr>
<td>$R_b$</td>
<td>20.03</td>
<td>20.03</td>
<td>20.024</td>
<td>20.024</td>
</tr>
<tr>
<td>m</td>
<td>0.06</td>
<td>0.08</td>
<td>0.18</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Bearing with two shims (0.024 mm):

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_p$</td>
<td>20.0355</td>
<td>20.0355</td>
<td>20.0355</td>
<td>20.0355</td>
</tr>
</tbody>
</table>

How it can be seen the range of preload values can vary in a notable way.

In the following the EKF identification procedure has been adopted under the hypothesis that the preload assumes the same value for all the pads of the bearing.

In particular, the actual value of $m$ has been evaluated through the identification of the clearances $C_p$ and $C_b$ since from them depend all the dimensionless quantities in eq. (1).

#### 4.1 Results

The experimental procedure has been conducted in this way:

- On the central disk, locked on the shaft rotor, an unbalance mass of known value, has been applied;
- During the run the angular velocity and the oil temperature have been monitored in order to verify that the steady state conditions have been reached;
- The $x(t)$ and $y(t)$ signals relative to a few shaft rotations have been acquired at a sample rate of 10 kHz using as a trigger the passage of the unbalance mass in the horizontal plane. A low pass filter at 400Hz has filtered these signals.

All the acquired signals have been stored to be used as measurement vector $z(t)$ in the subsequent identification procedure that consists in the solution of the differential equations system (5).

At each integration step, all the dimensionless quantities in eq. (1), together with the partial derivatives in eq. (6), have to be updated on the basis of the actual values of the parameters under identification.

In this particular case, as the acquired signals $x(t)$ and $y(t)$ are just some of the state variables, the measurements $H$ matrix in eq.(6) is constant and presents only two terms different from zero and equal to one in position 1,1 and 2,2.

Several attempts have been made in order to find the suitable values for the noise variance matrix $Q$ and $R$ since their values, unknown a priori, affect mainly the rate of convergence rather then the quality of the results [4, 5].

The fig. (6) reports an experimental orbit (black tick), described by the journal in the case of unbalanced rotor, starting from which has been evaluated the actual value of the geometrical preload that, substituted in the mathematical model, has given the numerical orbit superimposed (grey tick) in the same figure.
Fig. 6 Experimental and numerical journal orbits

The operating data for the case reported in the figure are:

- Pad offset: 0.6
- Number of shims per pad: 2
- Shaft angular speed = 7000 rpm
- Unbalancing mass = 9.7 g
- Unbalancing mass radius = 81.5 mm.
- Lubricant: ISO VG32
- Oil temperature = 35 °C
- Oil viscosity = 0.027 Pa s.

The orbits reported are described in the reference frame with origin in the bearing centre.

The actual $x_0, y_0$ coordinates of the bearing centre, with respect to the transducers, have been identified, but, as they depend on the position of the transducers themselves, they are not reported here because not meaningful.

With the identified values of $C_p = 0.06280$ mm and $C_b = 0.02698$ mm, the estimated value of the geometrical preload is $m = 0.57$.

How it can be noted, the comparison concerning the orbits’ dimension and position is satisfactory especially considering the simplicity of the mathematical model adopted.

A more sophisticated mathematical model, that adopts five different preload values for each pad of the bearing, has been developed, and, with the EKF identification procedure it has been possible to evaluate this different parameters.

Thanks to the peculiarity of the apparatus, that allows varying independently the five values of the pad preload, this technique has been tested and has successfully provided the attended results, even if it requires a more expensive computational time.

This feature could be particularly useful, for example, to detect the non-homogeneous wear of the pads.

5 CONCLUSION

An identification technique based on the extended Kalman filter applied to a system comprising a rigid rotor on tilting pad journal bearings has been described.

At the present the described procedure is applied off-line as the experimental data acquired on the test rig are elaborated in a subsequent moment.

An interesting prospective would be the development of a scheme of an on-line identification. The feature of such a procedure would consist in evaluating, during the operation, the actual values of the system parameters in order to prevent malfunctions. Moreover the procedure could result useful in order to apply some control strategies.

Nomenclature

- $A_j = \delta_j R / C_p$ = Pad inclination;
- $C_b = R_b - R$ = bearing clearance;
- $C_p = R_p - R$ = pad clearance;
- $D = 2R$ = bearing diameter;
- $E$ = rotor eccentricity;
- $F_x, F_y$ = hydrodynamic force component;
- $f_i = F_i / \sigma W$ = dimensionless hydrodynamic force component;
- $h$ = Oil film thickness;
- $h = \bar{h} / C_p$ = dimensionless oil film thickness;
- $\Gamma$ = Mass moment of inertia of pad around axial axis;
- $I = \bar{I} \omega^2 C_p / \sigma WR^2$ = dimensionless mass moment of inertia of pad around axial axis;
- $L$ = bearing length;
- $M$ = Rotor mass;
- $M = \bar{M} C_p \omega^2 / (\sigma W)$ = Dimensionless rotor mass;
- $m = 1 - (C_b / C_p)$ = geometrical preload;
- $R$ = journal radius;
- $R_b$ = bearing radius;
- $R_p$ = pad radius;
- $t$ = time;
- $T_j = \text{moment of the pressure force on the } j^{th} \text{ pad}$;
- $T_j = T_j / \sigma WR$ = dimensionless moment of the pressure force on the $j^{th}$ pad;
- $W$ = external load;
- $\bar{x}, \bar{y}, \bar{z}$ = Coordinates;
- $x = \bar{x} / C_p; y = \bar{y} / C_p; z = \bar{z} / L$ = dimensionless coord.;
- $x_0, y_0$ = Bearing centre coord. in the frame fixed to the transducers;
\( \beta \) = angular amplitude of pad;
\( \beta_p \) = angular distance of the pivot from the pad’s leading edge;
\( \beta_p/\beta \) = pad offset;
\( \delta \) = pad deflection angle;
\( \vartheta \) = Circumferential coordinate;
\( \vartheta_{pj} \) = Circumferential coordinate of the j\(^{th}\) pivot;
\( \mu \) = Dynamic viscosity;
\( \rho = E/C_p \) = dimensionless rotor unbalance;
\( \sigma = (\mu \omega RL/W)(R/C_p)^2 \) = Sommerfeld number;
\( \tau = \omega t \) = dimensionless time;
\( \omega \) = Angular rotor speed.

6 REFERENCES