ADHESION MODELS FOR WHEEL/RAIL CONTACT IN RAILWAYS

B. ALLOTTA, M. MALVEZZI, P. TONI
Department of Energetics “Sergio Stecco”, Section of Applied Mechanics, University of Florence, via Santa Marta n. 3, 50139 Firenze – ITALY; e-mail: malvezzi@mapp1.de.unifi.it, ptoni@mapp1.de.unifi.it, ben@sssup.it

SUMMARY
The forces and moments, which occur in the wheel/rail contact regions, have a dominant effect on the vehicle dynamics. Conventionally, the horizontal force exchanged between rail and wheel is expressed as the product of the vertical force acting on the wheel and the so-called adhesion coefficient. When the adhesion conditions between wheels and rail are degraded (for example in case of rain, fog, ice, dead leaves etc.) and the vehicle is accelerating or braking (i.e. if its acceleration is different from zero) pure rolling condition do not hold anymore, and macroscopic slips occurs on one or more of the wheels. Two similar phenomena occur during acceleration (in locomotives’ wheels) and braking (in all vehicles’ wheels), called slippage and skidding. This paper deals with the problem of identification of the parameters of adhesion suitable for describing wheel/rail interaction in trains, during braking. The adhesion in the wheel/rail contact area depends on a large number of factors; the first of these variables to be taken into consideration is slip. This work is devoted to find a model which correlates slip and adhesion coefficient.

Keywords: railways, wheel/rail contact, adherence, friction coefficient, and identification.

1 INTRODUCTION
The contact between wheel and rail, for its fundamental importance in railways, has been studied since the very beginning of railways. The experiences done by railway companies during the first half of the 20th century have supplied simple formulations based on friction coefficients.

Then, the growing speed has driven engineers' attention towards vehicle running stability which is influenced by the forces exchanged between wheel and rail. The contact problem has thus been more deeply investigated, by introducing more complex mathematical models.

By increasing the tangential force starting from zero, and fixed the remaining relevant parameters, two different phases can be distinguished in the wheel-rail contact phenomenon. In the first phase, named pseudo-sliding, due to elastic deformation of the two bodies, in the contact area, there is a first zone where relative slip occurs, and a second characterised by adhesion. In this phase contact forces may generate running instability: in design practice the incipient instability is evaluated by linearising the exchanged forces as in [1][2]. The dimension of the two zones depends upon the normal and tangential forces. With growing longitudinal forces the adhesion zone decreases till a limit situation where sliding occurs on the whole contact area. This is the second phase that may occur during braking and acceleration phases. In such running situations, the tangential forces depend upon the adhesion coefficient, which is a function of several parameters (normal load, sliding speed, temperature of the two bodies, contact geometry, weather conditions, presence of rain, snow, dead leaves, etc.) and the dependency on some of them may not be easy to express analytically.

The adhesion coefficient, which relates the vertical force on the wheel to the tangential force applied in the contact zone between rail and wheel, varies in a random manner. The research of a law that relates it to other directly measurable variables (train speed, wheel velocity etc.) is a difficult task. In this paper we report on some studies being done in collaboration between the University of Florence and TRENITALIA S.p.A. with the ambitious goal of obtaining a reliable model of adhesion to be used in an hardware-in-the-loop simulator devoted to the test of wheel slip protection systems (WSP) and odometry systems.

In Sec. 2 the state of the art in the formulation of the adhesion coefficient is summarised. In Sec. 3 we discuss some trends in the development of new test rigs devoted to the testing of railway subsystems which, in our opinion, justify the present research. In Sec. 4 we describe the simplified model of an isolated vehicle used to infer, by means of the method described in Sec. 5, adhesion data from existing results of running tests. In Sec. 6 we discuss the obtained results.

2 STATE OF THE ART

Due to the complexity of the phenomenon, with constant environmental conditions, it is commonly accepted to express adhesion as a function of sliding speed i.e. the difference between the vehicle speed and the tangential velocity of the wheel at the point of contact [3][4]:

$$\Delta v = v - R \omega,$$  \hspace{1cm} (1)

where $\Delta v$ is the relative speed, $v$ is the vehicle speed, $\omega$ is the angular wheel velocity and $R$ is the wheel contact radius.

In railway practice, instead considering $\mu$ as a function of relative speed, it is preferred to consider $\mu$ as a function of the train speed and the relative slip[5][6][7], given by the following expression:

$$v = \frac{v - R \omega}{\nu}$$ \hspace{1cm} (2)
The relative slip varies from 0 (when \( v = R\omega \), i.e. when pure rolling conditions are satisfied) and 1 (\( \omega = 0 \), i.e. the wheel is blocked).

![Figure 1: Adhesion coefficient/relative slip curve from [5].](image)

Some different theoretical sliding/adhesion curves have been proposed. For example, SCNF (France) [5], [6], [7] proposed a curve that relates the relative slip to the adhesion coefficient; its qualitative behavior is shown in Fig. 1. The curve presents two maximums, the first one is for small relative slip values (1.5%), while the second one is for relative slip values in the range between 5 and 25%.

This curve is obtained for a given value of train speed (constant). As the train speed increases, the curve moves to lower adhesion values, and the maximum value moves to higher slip value. The adhesion/slip curve depends also on the condition of the contact area, if this condition is degraded (for example in case of rain, fog etc.) the curve moves to lower values.

During a braking, the train speed decreases from an initial value to a final one. The adhesion coefficient does not vary on a curve like the one shown in Fig. 1, but on a “surface”, because both relative slip and train speed vary.

In order to fully describe the braking performance the braking performance of a given train, several families of curves as the one shown must be known, to account not only of varying train speed, but also for different adhesion conditions.

If the adhesion coefficient is related to the absolute slip [3] [4], the curve does not depend upon speed and a unique curve (for given adhesion conditions) is found.

### 3 PERSPECTIVE

The wheel-rail interaction model based on the adhesion coefficient represents a framework for a qualitative interpretation of the phenomenon. However, the task of interpreting experimental results of braking/traction tests by means of the model in Fig. 1 is not trivial. The problem is even more difficult to solve if "real-time" realistic simulators, including the wheel/rail interaction model in poor adhesion conditions, are needed. Such real-time realistic simulators have to be developed so to test in laboratory "runs" the behaviour of on board devices which are part of advanced train control and monitoring systems, thus reducing the cost of line tests. Among such train running control and monitoring systems we mention here the Italian system named SCMT [8] and the European one named ERTMS [9]. The latter will become a standard which the various railway companies of the European countries are currently testing (e.g. in the TRENITALIA-ALSTOM test site along the high speed Arezzo - Florence line) and intend to gradually introduce in all the railway network by the end of 2010, with the aim of increasing safety and service effectiveness. The on board devices of such systems perform in real-time odometric calculations to estimate travelled distance and train speed [10][11], mainly based on information coming from sensors located on some of the axles of the train (two, usually belonging to a locomotive). In this framework, the availability of reliable models to describe exchanged forces in braking and traction phases, in addition to the possibility of estimating the kinematic variables of the rolling stock in all the running conditions, including sliding and skidding, is of key importance. By the way, the availability of test rigs for on board devices, featuring a real-time model of a vehicle (or train) and the wheel-rail interaction model can be used, after validation, also for certification trials of wheel slip protection systems (WSP) commonly used in railway braking apparatus [12], as shown in Fig. 2.

![Figure 2: Test rig to evaluate performances of WSP systems.](image)
The experimental evaluation of wheel-rail friction is almost impossible to perform by using conventional test rigs, for the mechanics of contact between railway wheels and rollers used to simulate the presence of rail is an unacceptable deformation of phenomenological reality. On the one hand, rollers of so big diameter to be a good approximation of (zero) rail curvature should be so in practice not realizable. On the other hand, the instrumentation of a vehicle devoted to perform line tests aimed to study the wheel-rail contact is quite complex and expensive. A trade-off must be searched between the execution of a limited and targeted number of line tests, trying to simplify the instrumentation of the test vehicle by means of an accurate a priori analysis, and laboratory investigation, including theoretical study and analysis of experimental data. For the above mentioned reasons, the importance of the knowledge of the adhesion coefficient as a function of operational parameters is now and will certainly be in the next future of key importance for the new traffic control and management systems that railway companies want to introduce in Europe, for various reasons, including:

- to improve the performances and reliability of wheel slide protection systems (during braking);
- to improve the performance and reliability of anti skid traction control systems (during traction);
- to define design specifications for on board systems devoted to modify the way to drive trains, increasing both railway transport effectiveness and safety;
- to build simulators to test new on-board systems in order to perform their evaluation with reduction of expensive line tests.

4 MODEL DESCRIPTION

Each axles is characterized by:

- mass $m_i$
- moment of inertia $J_i$
- wheel radius $R$
- distance between the center of the wheel and the application point of braking force: $r$

On each axles a braking moment $M_{\beta} = c p_i$ is applied,

- $c$ is a constant;
- $p_i$ is the pressure in the brake cylinders.

The single axle (indicated by the index $i$) translation equilibrium equation can be written as follow:

$$ \sum_{i=1}^{4} F_{\beta i} - Q_i - T_i = 0 $$

$Q_i$ is the horizontal force transmitted from the vehicle to the wheel, $T_i$ is the horizontal force exchanged between the wheel and the rail, and $F_{\beta i}$ is the braking force, given by the following expression:

$$ F_{\beta i} = \frac{M_{\beta}}{r} = \frac{c}{r} p_i. $$

The horizontal acceleration is $\ddot{x} = -d$, the equilibrium equation can then be written as follow:

$$ -m_i \ddot{x} = Q_i - T_i - \frac{c}{r} p_i. $$

The rotation equilibrium equation of each axles (indicated by the index $i$) can be written as follow:

$$ J_i \ddot{\omega}_i = T_i R - M_{\beta} = T_i R - c p_i, $$

$$ T_i = \frac{J_i \ddot{\omega}_i}{R} + \frac{c}{R} p_i. $$

The term $\ddot{\omega}_i$ represent the angular acceleration of the wheel.

The horizontal vehicle translation equilibrium equation, obtained neglecting the aerodynamics and internal resistances, is:

$$ M_d = \sum_{i=1}^{4} (Q_i - F_{\beta i}) $$

from the equations (3) and (7) the following expression is obtained:

$$ M_d = \sum_{i=1}^{4} \left( T_i + \frac{c}{r} p_i - m_i \ddot{x} + \frac{c}{r} p_i \right) $$

The horizontal and vertical components of the resulting force exchanged between the rail and each wheel are related by the following expression

$$ T_i = \mu_i N_i $$

The adhesion coefficient can be evaluated from equation (6) and (9).

$$ \mu_i = \frac{1}{N_i} \left( \frac{J_i \ddot{\omega}_i}{R} + \frac{c}{R} p_i \right) $$

5 ADHESION COEFFICIENT IDENTIFICATION FROM EXPERIMENTAL DATA

In the experimental tests used for the identification the following data were available (Fig. 3):

- Absolute train speed, measured using an optical sensor;
- Wheel velocity, measured by encoder-type sensors placed on the wheel axes;
- Oil pressure in the brake cylinders.
Each information was sampled with a sampling period of 2.4 ms. The experimental tests were conducted using a single vehicle.

From wheel velocity information, the angular speed and acceleration were evaluated. The accelerations were evaluated using the finite difference method. The acceleration of the train was obtained from its speed, by derivation (using the finite difference method).

The absolute accelerations was filtered using a digital first order filter with a rise time of 2 s (i.e. a cut-off frequency of 0.5 Hz), while the angular accelerations were filtered using a filter with a cut-off frequency of 2 Hz.

The normal force acting on each bogie was evaluated using the following expression, in which the dynamic terms were neglected and the train deceleration were supposed constant:

\[
N_{i0} = \frac{M_i \cdot g}{2} \cdot d, \hspace{1cm} N_{i1} = \frac{M_i \cdot h}{2l} \cdot d, \]

where \(M_i\) is the case mass, \(g\) is the gravity acceleration, \(d\) is the train deceleration, \(l\) and \(h\) are respectively the horizontal distance between connection points between the case and the bogies and the vertical distance between the same points and the case center of gravity (as shown in Fig. 4). The normal force acting on each wheel was nearly:

\[
N_i = N_{i0} = \frac{N_{i1}}{2} + \frac{M_i \cdot g}{2} + M_a \cdot g, \hspace{1cm} N_i = N_{i1} = \frac{M_i \cdot g}{2} + M_a \cdot g, \]

where \(M_i\) and \(M_a\) are respectively the mass of the bogie and of the axles. The pull up angle were evaluated using the following expression:

\[
\vartheta = \frac{M_a \cdot h}{2k_2 l^2 d}. \]

Where \(k_2\) is the stiffness of the secondary suspension For each sample, the coefficient \(c\) defined in the previous section can be found from the following expression (obtained from equation (8)):

\[
c_a = \frac{1}{R} \sum_i J_i \omega_i \]

the \(c\) coefficient during a braking is represented on Fig. 6, as it can be seen, its variation is "small", so we supposed it to be a constant, whose value is the mean of the computed ones.

For each sample and for each axles the following values were evaluated:
- the adhesion coefficient \(\mu\) (using equation (10));
- the absolute speed \(v\);
- the absolute slip \(\Delta v\);
- the relative slip \(\nu = \Delta v/v\);
- the absolute slip derivative;
- the relative slip derivative.

The adhesion coefficient and the absolute speed during a braking test is shown in Fig. 6.
The adhesion coefficient, during a loss and a recovery of adhesion of an axles, has been evaluated. The identification zone has been chosen in order to:

- The train speed can be approximately considered constant;
- The derivative of the relative slip doesn’t change its sign.

Some results are shown in Fig. 8.

To evaluate the slip/adhesion curve, a grid where each element is characterized by a fixed absolute speed and a fixed absolute slip were defined. The size of the grid is 40x40, the speed varies from 0 to 40 m/s, while the absolute slip varies from 0 to 10 m/s. The difference between the reference speed of two adjacent elements was 1 m/s and the difference between the reference slip of two adjacent elements was 0.25 m/s. In each element the speed and the slip could then be considered constant.

For each element of the grid the corresponding \( \mu \) values were found and stored in a vector. A 3D matrix was obtained. The mean and the variance of each vector was computed and displayed on a 3D plot. The obtained surface was then sectioned, in order to examine the adhesion coefficient dependence on absolute speed and slip.

Some results are shown on Fig. 9 and discussed in the following section.

The same analysis was repeated using relative slip instead of absolute one. Some results are shown in Fig. 10. As it can be seen, in this case the adhesion coefficient depend also upon train speed, in particular, as the speed increase, the adhesion coefficient decreases.

### RESULTS

From the available experimental tests the coefficient that relates the pressure in the brake cylinders to the braking moment (that depends on the friction of the brake shoes and the adhesion coefficient between the rail and the wheel were evaluated. The coefficient that relates the pressure on the brake cylinders to the braking force is about constant during a braking. It has been also verified that the coefficient is nearly constant in different tests.

The analysis on the adhesion coefficient as function of the relative slip confirms these theoretical results:

- The adhesion coefficient depends also on absolute train speed, in particular, as the speed increase, the adhesion coefficient decrease;
- The adhesion coefficient is smaller if the derivative of the slip is positive (i.e. the axles is losing adhesion).

The analysis on adhesion coefficient as function of the absolute slip permits to conclude that:
The dependence on absolute train speed is not yet visible;
The adhesion coefficient decreases as the absolute slip increase.

7 CONCLUSION AND FUTURE WORK
This analysis was conducted using braking tests whose objective was the type approval of anti-skid devices. In these tests, even if the adhesion conditions were artificially degraded, the slip was controlled by these devices and was maintained in a narrow band. So only a part of the adhesion coefficient/absolute slip and adhesion coefficient/relative slip curves was identified.

Better results could be obtained if tests in which the sliding varied in a wider range (i.e. the relative slip varied from 0 to 1) were available.

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9 REFERENCES