SELF-REGULATING HYDROSTATIC PLATFORM

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SUMMARY
Hydrostatic axial systems are commonly supplied directly or by means of compensating devices; in addition, they are generally assumed either to rotate or to translate under unidirectional loads.

In this work a rotating and sliding hydrostatic system is investigated, subjected to bi-directional loads and with a self-regulating supply. Both the static and dynamic behaviours of the system have been analysed, obtaining a good agreement between theoretical and experimental results. The platform shows the high stiffness and dumping of the flow self-regulation and a greater efficiency than that one of the self-regulating bearing, under certain conditions.

Keywords: Lubrication, Hydrostatics, Self-regulation, Opposed pad, Dynamics.

1 INTRODUCTION
Hydrostatic bearings are characterized by a low frictional force \( F_N \) and a high stiffness \( K \). In particular the stiffness is very high if the bearing is directly fed by one pump. Still, when the bearings are more than one, the use a single pump supply system with fixed laminar-flow restrictors is to be preferred (for economic reasons) and it can compensate the loss of stiffness \[1\].

This is the case of opposed pad axial bearings under bi-directional load in Fig.1-a.

The pairs generally present either rotation or translation (in this case rectangular pad bearings are preferred).

In \[2\] the self-regulating opposed-pad circular bearing, shown in Fig.1-b, has been proposed.

![Figure 1: Usual and self-regulating systems: a- usual; b- and c- self-regulating hydrostatic systems](image)

Due to its particular shape this solution allows the use of one pump with the same "infinite stiffness" as the two pumps alternative.

In \[3\] the study has been extended to bearings with rectangular pads (Fig.1-c); in \[4\] to a complete slideway; in \[5\] to hydrostatic screw and nut assemblies, also supplied with constant pressure. In \[6\] the analysis has been faced of the dynamic behaviour of the system. A brief synthesis of these studies is reported in \[7\].

In this work the behaviour of a self-regulating bearing is investigated, with attention also to viscous resistance to the motion. The rotation and the simultaneous translation are enabled by a simple geometric variation, Fig.2. The dynamic performance is also studied. Experimental results, obtained with a model of this platform, are compared to the theoretical ones. The dynamic performance is also studied.

2 THEORETICAL RESULTS

2.1 Working principle

If \( R \) are the hydraulic resistances of the clearances, two of which act as variable restrictors (Fig.2-a), we have:

\[
R_i = R_{i1} + R_{i2}; \quad R_y = R_{y1} + R_{y2} \quad \text{and} \quad R_x = R_x
\]

If \( F_N = 0 \), the total flow rate \( Q \) is divided into two equal halves \( Q/2 \). Assuming \( r_1/r_2 = r_1/r_1 = r_1 \), we obtain:

\[
R_{y1} = R_{y1}, \quad R_{y1} = R_{y1} \quad \text{and} \quad R_{y1} = R_{y1} \quad (2)
\]

Even if \( F_N \neq 0 \), (1) and (2) are still valid and therefore \( Q = Q/2 + Q/2 \). In order to have the initial separation it must be \( r_1 > r_2 \).

In this platform, radius \( r_1 \) (shaft) and radii \( r_1 \) and \( r_2 \) are larger as regard as the current self-regulating bearings \[2\].

![Figure 2: Self-regulating platform: a- assembly; b- disk](image)
The self-regulation is now due to the annular relieves \((r_2 - r_1)\) and \((r_4 - r_1)\) of the inferior and superior moving disks.

The geometry of the system, so obtained, allows the rotation and the translation of the moving part; moving part is also very light. Further advantages will be point out after.

2.2 Static behaviour

Using usual conditions and mathematical approaches, supply pressure \(\bar{p}\) can be expressed by:

\[
\bar{p} = \frac{1}{2} \left[ \frac{1}{(1-\varepsilon)^2} + \frac{1}{(1+\varepsilon)^2} \right], \quad \bar{p} = -\int \frac{\mu}{\pi h_0} Q \ln r_u \]

with \(\varepsilon = (h_0 - h)/h_0\). It is also \(\bar{p} = p_u + p_\phi\) with

\[
p_u = \frac{1}{2} \left[ \frac{1}{(1-\varepsilon)^2} + \frac{1}{(1+\varepsilon)^2} \right], \quad p_\phi = \frac{1}{2} \left[ \frac{1}{(1-\varepsilon)^2} - \frac{1}{(1+\varepsilon)^2} \right].
\]

\(p_u\) and \(p_\phi\) are the superior and inferior recess pressures. The load capacity can be expressed by:

\[
F_N = F'_N F_N, \quad F_N = \frac{3\pi}{2} h_0 Q r_u^2, \quad F_N = \frac{1}{(1-\varepsilon)^2} - \frac{1}{(1+\varepsilon)^2},
\]

The stiffness of the bearing is \(K = K' \cdot K''\) with

\[
K' = \frac{3F_u}{h_o}, \quad K'' = \frac{F_N}{F_N}, \quad K'' = \frac{1}{(1-\varepsilon)^2} + \frac{1}{(1+\varepsilon)^2}.
\]

The pumping power is \(H_p = H'_{pr} H'_{pe} \), with

\[
H'_{pr} = \frac{3}{\pi} h_0 Q^2, \quad H'_{pe} = \ln r_u, \quad H_{pr} = \frac{1}{(1-\varepsilon)^2} + \frac{1}{(1+\varepsilon)^2}
\]

Solving \(\partial H'_{pr}/\partial r_u = 0\) we obtain, for the various \(r_u\), the “optimum values” \(r_u\) of \(r_u\), which minimize the pumping power \(H_{pr}\).

\(H_{pr}\) and \(r_u\) are shown in Fig.3-a, versus \(r_u\).

In Fig.3-b \(F_{N}\) and \(K'_c\) are plotted versus \(\varepsilon\) . We can express the quantities as functions of \(F_{N} = F_{N}/(F'_{N} F''_{N})\).

In Fig.4-a \(K'_c\) and in Fig.4-b \(H'_{pe}\) are plotted versus \(F_{N}\).

Note that \(F_{N}\) and \(Q\) as function of \(\bar{p}\) are

\[
F_{N} = -\frac{\pi}{4} r_2^2 F_N, \quad F_{N} = \frac{\pi}{6} h_0 \frac{1}{ln r_u} - \bar{p},
\]

so for a fixed \(F_{N}\), if \(r_u\) increases, \(\bar{p}\) and \(Q\) decrease.

The consequent increase of \(H_{pe} = \bar{p} Q\) compensates heavily the increase of the frictional power \(H_{pr}\). In presence of lack of parallelism \(\epsilon\), due also to an eccentric load, and evidenced by the angle \(\psi\) in Fig.5-a, an increase of \(r_a\) reduces not only \(Q\), but also the percentage of the losses of \(Q\), which are due to such errors. After all this system is advantageous not only because it allows roto-translation motion, but also because it requires generally rather low \(\bar{p}\), \(Q\) and \(H_{pr}\), and because is less sensitive to construction errors.

If the platform only rotates with angular velocity \(\omega\), frictional force \(F_{o}\) and frictional power \(H_{pr}\) are:

\[
F_{o} = \frac{4\pi}{3} h_0 \omega r_u^3 \left( \left(1 + \frac{r_u}{r_a} \right)^2 \right), \quad F_{o} = \frac{1}{1-\varepsilon} + \frac{1}{1+\varepsilon}
\]

\[
H_{o} = \frac{\pi}{2} h_0 \omega r_u^3 H_{pr} F_{o}, \quad H_{o} = \frac{1}{1-\varepsilon} \left( r_u \right)^3, \quad H_{o} = \frac{1}{1-\varepsilon} \left( r_u \right)^3
\]

If instead only translation with velocity \(u\) is present:

\[
F_{o} = 2\pi h_0 \frac{u}{r_u^2} F_{o}, \quad F_{o} = \left[ 1 + \left( \frac{r_u}{r_a} \right)^2 \right]^3, \quad H_{o} = \frac{1}{1-\varepsilon} \left( r_u \right)^3
\]

In Fig.3-a \(H_{o} = F_{o}\) is plotted versus \(r_u\), while in Fig.4-b \(H_{pe}\) and \(H_{pe} = F_{o}\) are plotted versus \(F_{N}\). 
In the case of combined motion (Fig.5), put \( v_r = \omega \cdot r \), the resultant velocity at the generic radius is:

\[
v = \left( u^2 + v^2 + 2uv \cos \theta \right)^{1/2}
\]

(13)

\[
\begin{align*}
\Delta h &= \frac{1}{4} \left( \frac{u^2}{r^2} + \frac{v^2}{r^2} + 2uv \cos \theta \right)^{1/2} \\
\end{align*}
\]

Since in the area of every gap the direct integration of \( v \) would involve the numerical calculation of elliptical integrals, it is convenient the integration of its series. This has consented to obtain the following relationship for the tangential force, with an error less than 1 %.

\[
F = \frac{8 \mu}{3 h_0} \left[ \left( \frac{r_1 + u}{\omega} \right) \left( \frac{\dot{r}_1 + \dot{u}}{\omega} \right) \left( \frac{\dot{r}_1 + \dot{u}}{\omega} \right) - \frac{1}{2} \left( \frac{r_1 + u}{\omega} \right) \left( \frac{\dot{r}_1 + \dot{u}}{\omega} \right) \left( \frac{\dot{r}_1 + \dot{u}}{\omega} \right) \right]
\]

(14)

\[
H_v = v \cdot F_v = v \cdot F' \cdot F'' \cdot F'''; \quad \text{in Fig.4-b it is also plotted} \quad H_{v_{\text{external}}} = F_v. \quad \text{If} \quad u = 0, \quad F_v \quad \text{has been reduced to} \quad F_{\text{internal}}; \quad \text{while for} \quad \omega = 0, \quad F_v = F_0. \quad \text{Obviously it is} \quad H_v < H_{\text{internal}} + H_{\text{external}}; \quad \text{for example for} \quad u' = v_r, \quad \text{Fig.5-b we have} \quad H_v = 0.62(H_{\text{internal}} + H_{\text{external}}).
\]

2.3 Dynamic behaviour

When the platform is subjected to a rapidly variable load

\( F_t(t) \) the system can be described by a mechanical analogy. The model is shown in Fig.6, where \( B_s \) represents the damping coefficient, while \( K \) represents the stiffness of the lubricant in the gap. The group \( B_r - K \) allows lubricant compressibility to be accounted for. If \( M \) is the moving mass and \( F_\text{external} \) the static force on the platform, we can express the law of motion as [6]:

\[
M \ddot{s} + B_\text{r} s = F_\text{external} + \delta F_N(1)
\]

(15)

\[
\delta F_s(p, \rho, \epsilon, \epsilon) = \frac{3 \pi}{2} \frac{\mu}{h_0} r_4,
\]

with

\[
B_s = B_{\text{r}} \left\{ -B_0, B_{\text{r}} - B_0 \right\}, \quad B_{\text{r}} = \left[ 1 - \left( \frac{r_{14}}{r_{14}} \right)^2 \right] \left( 1 - \left( \frac{r_{14}}{r_{14}} \right)^2 \right) \times \left( \ln \frac{r_{14}}{r_{14}} \right)^2.
\]

\[
B_{\text{r}} = \left( \frac{1 + \epsilon^{2}}{1 - \epsilon^{2}} \right)^{1/2} \times \left( \frac{1 - \epsilon^{2}}{1 + \epsilon^{2}} \right)^{1/2}.
\]

\[
F_{\text{external}} = \frac{6 \mu}{2 h_0} Q r_{14}^2 \left( 1 - \left( \frac{r_{14}}{r_{14}} \right)^2 \right) \times \left( \ln \frac{r_{14}}{r_{14}} \right)^2.
\]

\[
F_{\text{external}} = \frac{6 \mu}{2 h_0} Q r_{14}^2 \left( 1 - \left( \frac{r_{14}}{r_{14}} \right)^2 \right) \times \left( \ln \frac{r_{14}}{r_{14}} \right)^2
\]

\[
F_{\text{external}} = \frac{6 \mu}{2 h_0} Q r_{14}^2 \left( 1 - \left( \frac{r_{14}}{r_{14}} \right)^2 \right) \times \left( \ln \frac{r_{14}}{r_{14}} \right)^2.
\]

\[
F_{\text{external}} = \frac{6 \mu}{2 h_0} Q r_{14}^2 \left( 1 - \left( \frac{r_{14}}{r_{14}} \right)^2 \right) \times \left( \ln \frac{r_{14}}{r_{14}} \right)^2
\]
In studying the stability of the system, it can be assumed that the platform makes small vibrations around the static equilibrium position, so that it is possible to linearize the eqn. (15) and to apply the Laplace transform. If \( s \) is the Laplace operator, it results that:

\[
\delta \varepsilon \bigg( \frac{\partial}{\partial \varepsilon} - \frac{\partial}{\partial \varepsilon} \bigg) \frac{h_s}{M} + B_s h_s \delta \varepsilon \bigg( \frac{\partial}{\partial \varepsilon} \bigg) + B_{1s} \frac{d F_{Ne}}{d \varepsilon} \delta \varepsilon (s) + B_{1s} \frac{d F_{Ne}}{d \varepsilon} \delta (s) = \delta F_{Ne} (s)
\]

with \( B_s = \pi r_s^2 \). The total flow rate \( Q \):

\[
Q = \frac{p_{R_0}}{R_0} - B_s \frac{d F_{Ne}}{d \varepsilon} h_0 + \left( \frac{B_s}{B_1} \right) \frac{d p}{d \varepsilon},
\]

beside it may be put

\[
\frac{1}{K_d} = \frac{1}{B_s} \left( \frac{V}{K_d} + \frac{\partial V}{\partial p} \right)
\]

where \( K_d \) represents the lubricant stiffness, \( V \) the volume of the recess lubricant, \( V_t \) the volume of the relevant tubing; \( K_d \) is the apparent bulk modulus of the fluid, in which we can also consider the tubing elasticity.

\[
\frac{\delta \varepsilon}{\delta F_N} = 2 \left( h_s K_d \left[ 1 + 2 \left( \frac{\delta}{\omega_n} \right) + 4 \frac{B_s}{K_d} \frac{\delta}{\omega_n} - G_1 (1 - G_1) \right] \right)
\]

where

\[
\omega_n = (K_d/M)^{0.5}, \quad \zeta = \frac{1}{2} \frac{B_s B'}{K_d}
\]

In the case of \( F_{Ne} = 0 \), it is \( K' = 1 \) and \( G_3 = 0 \): we have a second order system with natural frequency \( \omega_n \) and damping coefficient \( \zeta \). It must be noted that \( B'B = \left[ B_s \left( -B_0 B_s + B_1 \right) \right] \); \( B* \) and \( B' \) are represented in Fig.8-a respectively versus \( r_s^2 \) and \( \epsilon \).

Fig.9-a represents the frequency response \( X' = e^{i \omega} K_d / F_0 = f(\omega/\omega_n) \) for certain values of \( \zeta = 2 \zeta \) [6], of a bearing excited by the force \( F = F_0 \cos(\omega t) \), in the generally assumed case of incompressible fluid.

Figure 8: a-Damping coefficients: \( B* \) versus radius ratio \( r_s^2 \); \( B' \) versus eccentricity \( \epsilon \); b-Coefficients \( G_1, G_2 \), \( G_3 \) versus eccentricity \( \epsilon \)

Obtaining \( \partial Q \) from the eqn. (14) and put \( \partial Q = 0 \), it results that:

\[
\lambda_p = \frac{\partial p}{\partial \varepsilon} = \frac{h_s K_d}{B'B^*} G_1 G_4, \quad \text{with} \quad G_4 = \frac{1 + G_2 \left( \delta / \omega_2 \right)}{1 + R \left( \delta / \omega_2 \right)}
\]

\[
\omega_1 = \frac{4}{3} \left( \frac{B_0}{B'} \right)^{2} R_0 \quad \text{and} \quad \omega_2 = \frac{K_d}{(B'B^*)^2 R_0}.
\]

If \( F_{Ne} = 0 \) it is \( K = K_0 \) and \( \lambda_p = 0 \). In the (18) \( \omega_1 \) and \( \omega_2 \) are the characteristic frequencies of the system, \( G_1 \) and \( G_2 \) are functions of \( \epsilon \) and they are plotted in Fig. 8-a. The block diagram is represented in Fig. 6-b. The transfer function of the system can be obtained by eqn.(15).

\[
\dot{\varepsilon} = \frac{h_s K_d}{B_0} \left[ 1 + 2 \left( \frac{\delta}{\omega_n} \right) + 4 \frac{B_s}{K_d} \frac{\delta}{\omega_n} - G_1 (1 - G_1) \right]
\]

For an arbitrary excitation, the response is given by the convolution integral [9], which in case of step function excitation can be expressed in this way:

\[
\epsilon(t) = \frac{F_0}{K_0} \left[ 1 - e^{-\frac{\epsilon}{\omega_n}} \cos \left( 1 - \zeta^2 \right) ^{0.5} \omega_n t - \phi \right],
\]
\[ \tan \psi = \frac{\zeta}{(1-\zeta^2)^{0.5}}. \]

Fig.9-b shows \(X\) versus \(\omega_n t\) for certain values of \(\zeta\).

3 EXPERIMENTAL APPARATUS AND RESULTS

The platform is shown in Fig.11, with some instruments. Radial dimensions (Fig.2) are: \(r_1=0.041\text{m}, r_2=0.045\text{m}, r_3=0.052\text{m}, r_4=0.057\text{m};\) so \(r_1/r_4 = 0.719\) and \(r_3/r_5 = 0.912\) (Fig.2).

In Fig. 10 we can see the insides of the system.

Fig. 10: Self-regulating hydrostatic platform: a-Coupled disks; b-Aerial view of the inside

The clearances in the centered position is \(h_0=2.10^{-4}\text{m}.\) Applied loads range is \(0-3000\text{N}, \varepsilon=0\) has been performed by counterbalance. The static and dynamic forces have been applied by weights which have been placed in the load-cylinder \(C\) shown in Fig.11.

A lubricant with viscosity \(\mu_20^\circ\text{C} = 0.417\text{ Nm}^{-2}\text{s}\) has been used; the test temperatures were \(22-55^\circ\text{C}\) with corresponding viscosity \(0.380-0.060\text{ Nm}^{-2}\text{s}\).

The system has been fed by an axial piston pump. Pressure and displacement transducers (Fig.11) and thermocouples have been used to detect pressures, clearances and temperatures during the tests.

In the case of combined motion, in order to evaluate the total frictional power, the load-cylinder (fixed to the platform) has been rolled without sliding on a rod placed at \(r = (r_1 + r_4)/2 = 0.049\text{m}\) from the axis of the platform (Fig.5).

Tests have been obviously performed for \(u=v_p\).

The results are presented in dimensionless form.

3.1 Static tests

In the Fig.12-b load capacity \(F'\text{N}\) and stiffness \(K'\) are plotted versus \(\varepsilon\)

A good connection between theoretical (solid lines) and experimental values has been found.

Figure 11: Self-regulating hydrostatic platform

Figure 12: a1-a2-Load capacity versus eccentricity; b1-b2- Stiffness versus eccentricity
In particular, the high stiffness of self-regulating systems has been confirmed for this platform too.

The few discrepancies may be ascribed to not perfect planarity and parallelism of the surfaces.

In Fig.13-a $K_{e}$ is plotted versus $F_N$; in Fig.13-b $H_{pe}$ and $H_{w} = H_{ue} = F_w$ are plotted versus $F_N$ too.

3.2 Dynamic tests

The dynamic behaviour of the system has been investigated by the fall of the load-cylinder C (Fig.11) on the disk D in order to have a step function excitation. In Fig.14 (filtered) diagrams of the displacements $X'$ versus $\omega_f t$ are represented for four values of damping coefficient $\xi$. The high values of $r_{\omega_f}$ avoid entirely cavitation phenomena in the lubricant films, which exist for the little values [10]. They confirm the high damping capacity of the self-regulating hydrostatic systems. A good agreement with results reported in [10] has been found too.

4 CONCLUSIONS

A flow self-regulating hydrostatic platform has been investigated, which can simultaneously rotate and translate for its particular shape.

The system is advantageous especially when low supply pressure and flow rate have to be used, and in presence of misalignment. A theoretical and experimental investigation on the static and dynamic behaviour of the system has been carried out.

Results confirm the validness of the self-regulated hydrostatic lubrication both in the static conditions, as concern the stiffness of the system and on the dynamic conditions, in particular as concern the damping characteristics.

5 REFERENCES