INTERACTION INVESTIGATION OF VIBRATING SURFACE AND THE TRANSPORTED DRY SUBSTANCES

G. BAURIENE
Kaunas University of Technology, Kestučio 27, LT-3004 Kaunas, LITHUANIA; e-mail: genbaur@vandenis.sc-uni.ktu.lt

A. BUBULIS
Kaunas University of Technology, Kestučio 27, LT-3004 Kaunas, LITHUANIA; e-mail: albub@mf.ktu.lt

K. PILKAUSKAS
Kaunas University of Technology, Mickevičiaus 37, LT-3000, Kaunas, LITHUANIA; e-mail: kepilk@tsc.ktu.lt

SUMMARY
The excitation of wave type deformation in a piezoceramic ring in its radial direction and the effect of these deformations on dry substance materials being in a ring-shaped channel is described in the paper. By controlling the parameters of vibrations, i.e. the excitation frequency of vibrations in a piezoceramic ring, the amplitude the preset dosing quantity and accuracy of dry substance material is achieved.

Keywords: traveling wave, piezoceramic ring, dry substance, dry friction, viscous friction.

1 INTRODUCTION
The methods of transportation of various materials with the use of elastic vibrations enable to develop a number of transportation mechanisms. Using the excitation of high frequency mechanical vibrations of various types (longitudinal, transverse, radial and other types of standing and traveling waves, using the capillary effect) in the piezoceramic operating links (pipes, channels of open or closed type the transportation of separate bodies, fluids or materials consisting of small particles can be performed. In references the diagrams of transportation mechanisms, enabling to transport fluids and materials of small particles are presented. In applications usually (in medicine, drug industry, power metallurgy) the micro technology enabling the precise transport of materials in small quantities is required (the speed of transport not greater than 100 grams/min). This requires the analysis of the transport of materials in definite small quantities. It is important to study the mathematical models for understanding the dynamics of transportation of small particle-type and fluid-type materials.

The original structure of a precision device for dosing and transportation of fluids and dry substances is described in references [1, 3, 4].

The operation of these mechanisms is based on the transformation of high frequency oscillations into directed movement of an output link. The most important part of the mechanism is its input link. In our case – it is a piezoceramic ring, as the output link is dry substance materials.

Analytically this is a complicated electromechanical system. In order to form its mathematical model one must know its mechanical and electrical parameters and high frequency vibrotransportation methods. The most difficult task is to determine the relationship between the exciter, and the output link i.e. between the system with distributed parameters and the one which motion is expressed by a conventional differential equation. Therefore, the dynamics of such system with the ring-shaped exciter and dynamic processes in it were studied by means of the method of finite elements. Free and forced oscillations of a ring-shaped piezoceramic element were analysed.

2 DYNAMIC PROCESS IN PIEZOCERAMIC RING
Dynamic processes taking place in a piezoceramic ring-shaped exciter were analysed by the finite element method [2] i.e. free and forced oscillations of the piezoceramic exciter were investigated. High frequency vibrations of the piezoceramic ring-shaped element were excited by connecting its electrodes to single and multi-phase power sources.

Piezoceramic ring made of piezoceramics CTS-19 (lead circonate) was used for calculations and investigation. It was divided into 20 finite elements with 40 modes having the ring-shaped sectors with the following parameters: \( R_1=0.011 \text{ m}, R_2=0.013 \text{ m} \) - inner and outer radii of the ring; \( \gamma=3\cdot10^7 \text{ N/m}^2 \) – elasticity modulus of piezoceramics; \( \rho=7200 \text{ N/m}^3 \) – density of piezoceramics [2].

![Figure 1.](image-url)
Different forms of free oscillations can be excited in the piezoceramic rings (Fig. 1). These forms of free oscillations are determined by the number of mode lines which can be \( n = 2, 3, 4 \). The radial displacement along these mode lines is zero.

Four forms of radial oscillations were obtained with the following frequencies:

\[
\begin{align*}
\omega_1 &= 53996 \text{ s}^{-1} ; \\
\omega_2 &= 8639 \text{ s}^{-1} ; \\
\omega_3 &= 11652 \text{ s}^{-1} ; \\
\omega_4 &= 15149 \text{ s}^{-1}.
\end{align*}
\]

The theoretical frequency of the first form of free oscillations can be obtained from the following expression \([1]\).

\[
(\omega_0)_{\text{theoretical}} = \sqrt{\frac{\gamma}{a^2 \rho}}, \quad \text{where} \quad a = \frac{R_2 - R_1}{2}.
\]

From the calculation results it was obtained that \( \omega_0 = 5380 \text{ s}^{-1} \) and close to \((\omega_0)_{\text{theoretical}}\).

For the analysis of forced oscillations the piezoceramic ring made of the same material (CTS-19) was used. It was divided into 20 finite elements and the second \((n=2)\) form of free oscillations \((\omega_2 = 8639 \text{ s}^{-1})\) was chosen.

The matrix differential equations of motion of any sector point can be written as follows:

\[
\begin{bmatrix} M \end{bmatrix} \{\delta\} + \begin{bmatrix} K \end{bmatrix} \{\delta\} + \begin{bmatrix} C \end{bmatrix} \{\delta\} = \{f_j'(t)\},
\]

where \([M]\) – matrix of masses, \([K]\) – matrix evaluating damping, \([C]\) – stiffness matrix, \(\{f_j'(t)\}\) – vector of forces due to electrical field, \(i\) – member of finite elements, \(j\) – phase, \(\{\delta\}\) – shift vector of mode points which is equal

\[
\{\delta\} = \begin{bmatrix} \delta_\rho \\ \delta_r \end{bmatrix},
\]

where \(\delta_\rho\) – displacement in radial direction, \(\delta_r\) – displacement in tangential direction.

The force vector

\[
\{f_j'(t)\} = A \sin(\omega_k t + \phi_k),
\]

where \(t\) – time, \(A\) – amplitude of oscillations, \(\omega_k\) – frequency of the second form of oscillations, \(\phi_k\) – phase shift.

When piezoceramic ring is excited by single-phase power source the oscillations in the ring do not have wave character (Fig. 2).

When it is excited by three phase power source the radial wave type oscillations are obtained (Fig. 3). The conclusions can be made: in order to excite travelling wave oscillations in a piezoceramic ring the three-phase power source is necessary.

### 3 DYNAMICAL PROCESS IN A CLOSED RING-SHAPED CHANNEL

The investigation of a supply and dosing device for fine particle dry substance material is presented in case when the particles pressed together can move in linear direction one with reference to the other. The investigation is performed under the assumption that dry friction forces appear between the particles.

As an example a ring-shaped closed channel rectangular in cross-section is investigated, when one wall 2 of it is excited by high frequency oscillations. The source of oscillations is piezoceramic ring 3 fed from signal generator so that the forced vibrations in it are excited and with a phase shift in several different points what creates the conditions for a traveling wave type deformations to appear in the channel.

![Figure 4: Experimental test rig of the dosing device prototype](image)
The forced vibrations of the ring-shaped channel are excited by a force, which can be expressed by the following equation:

\[ p(\varphi, t) = \sum_{j=1}^{n} c_j \delta(\varphi - \varphi_j) \sin(\alpha - \Theta_j) \]  

(5)

Where \( p(\varphi, t) \) – the force acting in points \( n \); \( c_j \) – the force amplitude; \( \Theta_j \) – phase shift; \( j = 1, 2, 3, \ldots \); \( t \) – time; \( \omega \) – frequency; delta function \( \delta(\varphi - \varphi_j) \) is determined in the interval \([ -\pi, \pi ]\).

The solution for radial deformations to be determined is expressed as follows:

\[ u(\varphi, t) = -\frac{R^3}{EJ} \sum_{k=1}^{\infty} \frac{k^2}{\omega^2 (k^2 + 1) - k^2 (k^2 - 1)^2} \times \sum_{j=1}^{n} c_j \sin(\alpha - \Theta_j) \cos(\varphi - \varphi_j). \]  

(6)

Where \( R \) – radius of the ring, \( E \) – modulus of elasticity, \( J \) – moment of inertia, \( h = A \gamma g / EJ \), \( A \) – cross-sectional area of the ring, \( \gamma \) - density, \( g \) – acceleration of gravity, \( k = 1, 2, 3, \ldots \).

Natural frequency of the ring in its plane:

\[ \omega_n = \frac{k(k^2 - 1)}{\sqrt{h(k^2 + 1)}} \]  

(7)

If \( \omega = \omega_k \) then the linear velocity of the created travelling wave:

\[ \omega = \frac{R(k^2 - 1)}{\sqrt{h(k^2 + 1)}} \]  

(8)

When the force \( p(\varphi, t) \) acts on the ring in two points \( n = 2 \), linear velocity of the traveling wave \( v_2 = 1.34R / \sqrt{h} \), when \( n = 3, \ v_3 = 2.33R / \sqrt{h}, \ n = 4, \ v_4 = 3.64R / \sqrt{h} \). That is the deformation amplitude and traveling wave velocity of the channel walls can be calculated analytically. From this follows that particle number of the substance – dose directly depends on such parameters of the excited vibrations in the channel as frequency, amplitude and excitation duration as well. The frequency as the resonance parameter of the piezoelement is fixed (for certain element) and amplitude adjustment does not satisfy dosing accuracy requirement, so the excitation duration adjustment remains.

Principle schematics of the dosing device is shown in Fig. 5.

It consists of a rectangular ring-shaped channel 1, powder reservoir 3 with an inlet pipe and metal powder 4 in it and piezoceramic ring 2 attached to the inner wall of the channel. When electrical signal is fed from signal generator to the electrodes of the ring it starts vibrating and wave type deformations appear in the channel. Under the effect of these deformations the particles of metal powder in the channel are dosed. By controlling the excitation duration the supply amount of the metal powder can be adjusted. Experimentally were determined the motion characteristics of the particles of dry substances.

**Figure 5:** Principle schematics of the dosing device: 1 – ring-shaped channel, 2- piezoceramic ring, 3 - tank with the dry substance to be dosed.

**Figure 6:** The characteristics of micro dosing device for dry substance supply: \( Q \) – the material amount per time unit versus the amplitude of vibration excitation, \( U(V) \) – the accuracy of material supply \( \delta \) versus the excitation amplitude of vibrations.

Fig. 6 shows the efficiency, i.e. the rate of metal powder amount per time \( Q \) and the accuracy of supply \( \delta \) versus amplitude of the excited deformation. Two type metal powder were dosed: \( \Delta \) – (Cr, C2) and \( \bullet \) (IIT12H-01) – their filling weight 3 g/cm\(^3\) and 4.5 g/cm\(^3\).
4 CONCLUSIONS

Deformations (of a travelling wave type) excited in piezoceramic rings and transferred to structural elements (like metal ring-shaped channel) in dry friction or viscous friction ways interacting with a material can be applied for the transportation and precision dosing of fluids, dry substances and hard bodies as well as the realization of separation process of dry substances.

Theoretical investigations of piezoceramic rings by the method of finite elements, when they were excited by electrical pulses revealed the wave type deformations, which can be obtained, and their radial forms as well. Experimental results in the designed micro dosing prototype showed the possibility to dose dry substances (metal powder) by small doses and with high precision.

5 REFERENCES