1 INTRODUCTION

This paper examines the role of lubrication played in mixed friction. Under mixed friction we mean such regime when its integral characteristics are composed of liquid, boundary and unlubricated friction components which simultaneously exist in a tribozone in different areas.

Friction zone is an area of conjugation of couple elements receiving external load. It consists of sites of real area soft contact (RAC), at which the regime of boundary or unlubricated friction is realised, as well as spaces in between which are filled with lubrication. It is supposed that between RAC the latter has volumetric and adhesion properties and also can receive a part of external load due to the appearance of hydrodynamic and hydrostatic effect, which occurs under relative displacement of friction surfaces and their flow between RAC. Such regime is sometimes referred as a semi-liquid film friction.

Mathematical description of lubrication flow along curvy channels between RAC presents the greatest interest as it should differ from the classical one which is used in the hydrodynamic theory of lubrication.

Rather limited number of works deal with the description of mixed friction. Here we should mention the works of L.Gumbel, A.M. Ertel, A.S. Akhmatov, E. Kingeboum, K. Komvopoulos, N. Sack and N.P. Su, S.V. Ventsel, I.K. Appeldorn, V.A. Koudinov, I.V. Kragelsky, etc. All the works mentioned touch upon different aspects of mixed friction or its separate components. However, mathematical description of this process doesn’t exist so far. The present work attempts to fill in this gap.

2 MATHEMATICAL MODEL OF MIXED FRICTION

In defining the mathematical model, we admit that the force of friction in tribozone is an additive function, i.e. it is a sum of friction forces of elementary areas of the tribozone (Fig.1). Let us take a radial plain bearing as an example.

An absolutely hard journal having a regular roughness modelled by triangular prisms (in future the form of roughnesses can be generalised) is introduced into a rigid-plastic bush to a definite depth under the action of a pre-set radial load \( P \) (Fig.2). In this, the journal and the bush (insertion) contact over certain surface consisting of separate areas of real contact, RAC. The radial load \( P \) to the bearing is partially received by RAC and partially by the lubrication located between them.
When the journal rotates relative to the bush, the lubrication moves along the curvy channels between RAC similar to the movement (filtration) of liquid in porous medium creating at the same time some kind of hydrodynamic pressure $p$. Let us consider that the parameters of surface roughnesses, their elastic, plastic and thermophysical properties are set.

Besides, rheological lubrication properties, as well as external effects upon the bearing: external radial load $P$, journal rotation speed relative to the bush $u$ and environmental temperature $T$ are pre-set. All the enumerated parameters can be also given as functions of time and coordinates. It is necessary to define the portion of friction force realised at RAC, lifting power of the layer is created by a forced pressure gradient created at the expense of speed of relative displacement of friction surfaces can be negligible. On the basis of the above statements, to develop a mathematical model its restrictions are confined with i.e. its flow is described by the laws of hydrodynamics.

The flow between RAC satisfactorily must be laminar. In porous medium it is described by the fundamental law of flow of homogeneous liquids in porous media (Darcy’s law) with Reynolds’ numbers $Re$ 500 [1].

Lubrication must wet friction surfaces, i.e. be kept upon them by adhesion forces.

4. Friction force between the journal and the bush is equal to the sum of elementary friction forces at RAC and the film between them (additivity).

On the basis of the above statements, to develop a mathematical model we shall consider friction zone to be a porous medium and use the main notions of homogeneous liquids filtration theory through porous media. In this, roughnesses of friction surfaces can be of either regular, or random form. In contrast to classical description of liquid flow in porous medium by means of filtration theory, in a boundary layer of friction zone the pressure gradient is created due to liquid movement in a porous layer similar to hydrodynamic bearings, rather than a liquid moves due to the creation of pressure gradient. This is the reverse problem of filtration theory.

However, in considering hydrostatic supports, when the lifting power of the layer is created by a forced pressure of lubrication fed into the friction zone, we deal with the direct problem of filtration theory if the pressure gradient created at the expense of speed of relative displacement of friction surfaces can be negligible.

Fundamental law of liquid flow in a porous medium is the law of Darcy [2]:

$$
\mathbf{v}_i = -\frac{k_i}{\mu} \left( \nabla \phi + \mathbf{g}_i \right),
$$

where $i = x, y, z$.

Permeability coefficient $k$ in liquid filtration theory in porous media is most often expressed through Sallivan’s formula [1]:

$$
k_i = \frac{c \Pi^2 \theta}{S^2 (\theta - \Pi F_i)}.
$$

The system of equations (1) should be supplemented by the equation of continuity of liquid flow in a porous medium:

$$
\frac{\partial \rho}{\partial t} + \nabla \cdot \left( \rho \mathbf{v} \right) = 0,
$$

where

\begin{itemize}
  \item $\Pi$ - average volumetric porosity of the medium;
  \item $\rho$ - material density
\end{itemize}

In a thin lubrication film the influence of mass forces $G$ as compared to viscosity forces can be neglected. In this case the differential equation of filtration can be written as:

$$
\frac{\partial \rho}{\partial t} - \frac{\partial}{\partial x} \left( \rho \mathbf{v} \right) = 0
$$

where $\mu$ - dynamic viscosity coefficient.

Considering non-isothermal flow of liquid (lubrication) in a porous medium, it is necessary to use the heat transfer equation:

$$
\frac{\partial}{\partial t} \left( \rho c \mathbf{v} \right) + \nabla \cdot \left( \rho c \mathbf{v} \mathbf{v} \right) = \nabla \cdot \left( \lambda \nabla T \right) + q V
$$

where $\lambda$ and $c$ - thermal conductivity ratio and specific thermal capacity, respectively; $q V$ - power of internal volumetric heat source; with conformation boundary condition where $q V = \tau_c U_0$.

For the forces acting upon the bearing, we can write down the equilibrium condition:

$$
\tau_c = \mu \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) + \tau_c.
$$

For a porous medium the form of the resistance function was suggested by M.A.Sattarov [3]:

$$
\tau_c = \mu N(z).
$$

However, we must stress that according to the theory of dimensions, $N(z)$ is a pressure referred to dynamic viscosity.

The same work gives two types of function $N(z)$:

$$
N(z) = a m_1 z e^{m_1 z} \quad \text{and} \quad N(z) = a \left( \xi / R_k \right)^{m_1},
$$

where $a$ and $m_1$ - coefficients and constants of surface effect in liquid volume and at the border of contact, respectively.

As in description of lubrication flow in a rough contact we have to operate with speeds averaged over the porous medium thickness, it is necessary to define the average displacement strain over the layer thickness with the account of additional resistance.

The final expression for the average displacement shear strength in lubrication film of porous contact can be written down in the following way:

$$
\tau_c = -\frac{\mu}{h} \left( \frac{\mathbf{U}_0}{h} + a \varphi \left( \frac{h}{n - 1} \cdot \frac{1}{m^2} \right) + \frac{\gamma}{m^2} \right)
$$

Hydrodynamic component of the friction moment between the journal and the bush and the portion $P_H$ of
radial force P received by lubrication is defined from (12) by the expression
\[ M_{\text{a}} = \int_{r_{0}}^{r_{a}} \tau_{r} \, \text{d}s \]  
(8)
\[ P_{\text{a}} = \int_{r_{a}}^{r} \rho \, \text{d}s \]  
(9)

The equations (1) - (9) are forming mathematical model of liquid friction between rough contact surfaces. The calculation method had been given in paper [4].

3 SOLUTION ALGORITHM AND METHODS.

As the bearing surfaces making the porous medium move relative to each other at a velocity \( U_0 \), Eqs. (1) and (4) of the model need correction.

To relate the average lubricant flow velocity in the clearance during the relative motion of the surfaces to the linear velocity \( U_0 \), we consider the plane flow of liquid in the direction \( x \) in a slot with the characteristic linear dimension \( \zeta \). Basing upon the Navier-Stokes equations for mobile slot walls we have,
\[ \bar{U}_{y} = -\xi x_{y} \frac{\partial p}{\partial x} \frac{U_{x}}{2} \]  
\text{(10)}

Hence the average lubricant velocity in the porous medium with mobile walls,
\[ v_{s} = \bar{U}_{s} = \frac{K_{e}}{\mu \sigma} \frac{\partial p}{\partial x} \frac{U_{x}}{2} \]  
\text{(11)}

For the radial bearing, \( \Pi = \Pi(\phi) \) due to eccentricity and the difference between the radii of the journal and the bush; \( U_{b} = U_{a}(t), \Pi = \Pi(\phi,t) \) under nonstationary conditions. Consequently, Eq. (5) of the mathematical model for this boundary value problem can be written as,
\[ \frac{1}{\mu} \frac{\partial}{\partial \phi} \left( \frac{K_{e}}{\mu \sigma} \frac{\partial p}{\partial x} U_{x} \right) + \frac{\partial}{\partial y} \left( \frac{K_{e}}{\mu \sigma} \frac{\partial p}{\partial y} \right) = 0 \]  
\text{(12)}

Here \( K_{e} = K(\phi, t), \Pi = \Pi(\phi, t), U_{b} = U(t), K_{y} = K(y, t), \mu = \mu(p, T) \), which is equivalent to \( \mu = \mu(\phi, y) \). For simplicity, we study the case of the stationary lubricant flow, when \( K_{e} = K(\phi), \Pi = \Pi(\phi), K_{y} = K(y), \mu = \mu(\phi, y) \).

When solving Eq.(12), we give explicit expressions of the functions \( K_{e}, K_{y}, \Pi, \mu \) and the boundary conditions.

On account of Sullivan’s formula given in [1],
\[ K_{e} = \frac{c \, \gamma^{2} \, \theta}{S^{2} \, n - I} \]  
\text{(13a)}

where \( K_{e} \) is the longitudinal roughness constant, \( C \) is the Kozeny constant, \( S \) is the Karman specific surface (or the total pore surface to porous medium volume ratio), \( \theta \) is orientation factor.

\[ \Pi = \frac{V_{s}}{V_{s} + V_{m}} \]  
\text{(14)}

where \( V_{s} \) and \( V_{m} \) are the volume of space between the micro-hills and the volume of micro-hills respectively
\[ V_{s} = b_{s}h(\phi) \]  
\text{(15)}

The variable thickness of the clearance modelled by the porous medium is,
\[ h(\phi) = h_{s} + h_{c} - \frac{\xi x_{y} \cos \phi}{2 \xi} \]  
\text{(16)}

The amount of the free specific surface \( S \) is given for the regular triangular profile by the formula (Fig.2b)
\[ S = \frac{(1 - \frac{1}{\xi \sin K})^{2} - (1 - \sqrt{2} \xi \sin K)}{\xi \sin K} \]  
\text{(17)}

The nomenclature is given in Fig. 2. On account of Ref. [5], we use the dependence of lubricant viscosity on temperature and pressure as,
\[ \mu = \exp \left( a_{s} + \frac{b_{s} + c_{s} \pi}{T - T^{\infty}} \right) \]  
\text{(18)}

where \( a, b \) and \( c \) are empirical coefficients for the type of the lubricant used, \( T \) and \( T^{\infty} \) are current temperature and congelation temperature respectively. Note that, if one deals with non-Newtonian lubricants, Eq.(18) can be different.

To determine \( T \) in (18) as a coordinate function, one is to solve Eq. (4) of this model, with the following boundary conditions,
\[ T = T_{0}(r, \phi, y) \]  
\text{a)}

\[ \lambda_{s} \frac{\partial T}{\partial x} + \lambda_{m} \frac{\partial T}{\partial x} + q_{v} = 0 \]  
\text{b)}

\[ \lambda_{s} \frac{\partial T}{\partial x} + \lambda_{m} \frac{\partial T}{\partial y} + h_{t}(r - T^{\infty}) = 0 \]  
\text{c)}

where \( T_{0} \) and \( T^{\infty} \) are given and ambient temperatures respectively; \( l \) and \( m \) are the direction cosines of the external normal to the contact boundary; \( h_{t} \) is the factor of heat exchange with the environment, \( q_{v} \) heat density.

The quantity \( q_{v} \) is here assumed to be equal to the specific power of friction, which consists of the respective powers on the RAC, and in the lubricant film between the RAC
\[ q_{v} = \tau_{s} U_{s} + \tau_{e} U_{e} \]  
\text{(20)}

where \( \tau_{s} \) is the tangential component of the surface stress on each micro-hill on the RAC; \( \tau_{e} \) is the stress tensor component in the lubricant film between the RAC, which has the same direction as the relative displacement of the surfaces; \( U_{0} \) is relative surface displacement velocity.

The amount of RAC is determined by the formula
\[ \sum_{RAC} = P \frac{c_{\sigma}}{c_{\sigma}} \]  
\text{(21)}

where \( P \) is normal loading on the bearing; \( c_{\sigma} \) is the factor allowing for the mechanical properties of the bush material; \( c_{\sigma} = 2.7...3.1 \); \( \sigma_{T} \) is yield stress for the bush material.

The value of \( c_{k} \) is found by the formula of Ref. [4],
\[ \tau_{w} = \mu \left( \frac{\partial U}{\partial x} + \frac{\partial U}{\partial z} + \tau_{0} \right), \quad (22) \]

where \( \tau_{0} \) is additional shear strength caused by the sinuosity of the canals forming the porous medium.

Since this model deals with the average velocity of the lubricant flowing in the canals of the porous medium, Eq. (22) is reduced to the following form (taking account of the stress \( \tau_{0} \) allowing for the capillary effects in the flow of liquid in the porous medium),

\[ \tau_{w} = \frac{1}{m} \left( \tau_{w} d x = - \mu \left( \frac{U_{0}}{h_{0}} + a e^{-\pi} \right) \left[ \frac{n_{1}}{m} - \frac{1}{m} \right] + \frac{a}{m^{2}} \right). \quad (22a) \]

Here \( a \) and \( m \) are factors characterizing the capillary properties of the lubricant and the porous medium. The value of \( \tau_{0} \) in the lubricant film is usually small, as compared to the shear stress (according to our analysis, 2 to 3 orders of magnitude smaller), therefore it can be neglected. Then,

\[ \tau_{w} = \mu \frac{U_{0}}{h_{0}}, \quad (22b) \]

where \( h_{0} \) is calculated by Eq. (16).

If the values of \( \tau_{k} \) and \( \tau_{pr} \) are multiplied by the radius \( r_{2} \) of the pitch, the strain and fluid components of the friction moment are obtained. By adding the products and integrating them with respect to the friction zone area, we determine the friction moment in the bearing,

\[ \frac{1}{2} \tau_{s} = \eta_{s} \int_{S} \tau_{w} d s + \int_{S} \tau_{w} d s, \quad (23) \]

where \( \eta_{s} \) is the relation of the real area of contact to the geometric area of contact.

The value of the hydrodynamic component of the response of the lubricant film between the RAC is determined after finding the function of pressure \( p \) in the lubricant film over the friction zone area,

\[ P_{n} = \xi - \eta_{m} \int_{S} P_{d} d S, \quad (24) \]

The solution procedure starts from the specification and determination of all the parameters required for the solution. The block - diagram of the algorithm is given in Fig. 3. The differential equation of heat transfer (4), with the boundary conditions (19), is solved by the finite element method within the “Thermos” package developed by G. P. Gasilov. The differential equation of filtration (12) is solved by the Newtonian method in accordance with the programme developed. The mean shear strength \( \tau_{k} \) for one hill - hollow pair of the strain component of the response \( P_{d} \) of the bush, and the intensity of wear \( J \) are calculated by the programmes for calculating un lubricated friction parameters, which are found by the formulae given in [4].

The calculation made by this algorithm results in the fulfilment of the equilibrium condition with a given discrepancy (degree of accuracy) \( \varepsilon \).

\[ P = (P_{d} + P_{n}) \cos \varphi_{0} + (F_{a} + F_{b}) \sin \varphi_{0}, \quad (25) \]

where \( \varphi_{0} = \arctan \frac{F_{a}}{F_{b} + P_{b}}. \)

This condition is achieved at a definite number of iterations \( P_{d} \) and \( P_{n} \).

Fig. 3: The solution algorithm for the boundary value problem of finding the parameters of mixed friction for the radial plain bearing.

The iteration process converges fairly quickly even at small values of \( \xi \) - about 1 to 2 \% of \( P \). After the convergence, i.e., after the condition of Eq. (25) is observed, the total moment and factor of friction between the journal and the bush and its components, as well as the intensity of wear under given conditions, are calculated and the isotherms in the bush and the conditional epure of pressure distribution in the lubricant film between the RAC are constructed.

4 REFERENCES