AN IDENTIFICATION TECHNIQUE FOR THE
CONFIGURATION STATE OF STATICALLY
INDETERMINATE ROTOR BEARING SYSTEMS

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SUMMARY
In statically indeterminate rotor bearing systems involving hydrodynamic bearings, the relative lateral alignment of the bearing housings, i.e., the system configuration state, has a significant influence on the system stability and its vibration behaviour. This paper deals with a general identification procedure for determining the configuration state of any statically indeterminate system with hydrodynamic bearings. The technique is illustrated via numerical experiments using a fictitious four bearing system; and utilises a knowledge of the relative motion of the flexible rotors in their respective housing and of the absolute motion of the flexibly supported bearing housings, information which is now frequently available in existing turbomachinery installations. It is shown that acceptable configuration state identification is obtained even with 5% measurement uncertainty, here simulated by truncating all numerical ‘measurements’ to two significant digits. Since 5% measurement uncertainty is attainable with present measurement instrumentation, the identification technique is expected to be practically realisable.

Keywords: Lateral Misalignment, Identification, Configuration State, Rotor Bearing Systems, Hydrodynamic Journal Bearings.

1 INTRODUCTION
In statically indeterminate rotor bearing systems involving hydrodynamic bearings, the relative lateral location of the bearing housings, i.e., the system configuration state, has a significant influence on the system stability and its vibrational behaviour. This is because the stiffness and damping afforded by these bearings depend on the bearing reaction forces which, in turn, for the fixed bearing dimensions, depend on the relative alignment of the bearing housings with respect to each other, as well as on operating conditions such as lubricant viscosity, lubricant supply and exhaust pressures, operating speed and unbalance state. In practice, the configuration state at running conditions is frequently unknown because of thermal considerations and/or foundation settlement; and a feasible means for evaluating this state is highly desirable.

This influence of the configuration state on the system vibration behaviour is well recognised [1,2,3] and for turbomachinery vibration analysis and control, it is necessary to know the actual configuration state. Ref. [2] developed an identification technique for rigidly supported rotor-bearing systems, though experiments provided qualitative rather than quantitative agreement. The technique to be presented here, while similar to ref. [2], is quite different in its implementation; and by including gyroscopic effects and flexible pedestal supports, is more closely aligned to practical turbomachinery installations.

2 NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i$</td>
<td>location of $i$’th unloaded bearing housing</td>
</tr>
<tr>
<td>$b_i$</td>
<td>zero frequency displacement of $i$’th bearing housings</td>
</tr>
<tr>
<td>$C, C$</td>
<td>damping and gyroscopic matrix, element therein</td>
</tr>
<tr>
<td>$e_i$</td>
<td>zero frequency journal eccentricity at $i$’th bearing</td>
</tr>
<tr>
<td>$F$</td>
<td>force vector</td>
</tr>
<tr>
<td>$f$</td>
<td>force frequency component vector</td>
</tr>
<tr>
<td>$f, f$</td>
<td>force frequency component vector, element therein</td>
</tr>
<tr>
<td>$K, K$</td>
<td>stiffness matrix, element therein</td>
</tr>
<tr>
<td>$k$</td>
<td>order of harmonic; $k = 0, 1, \ldots, n$</td>
</tr>
<tr>
<td>$M, M$</td>
<td>mass matrix, element therein</td>
</tr>
<tr>
<td>$n$</td>
<td>highest order harmonic</td>
</tr>
<tr>
<td>$P, Q, R, S$</td>
<td>partition matrices defined by eqn (9)</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$X$</td>
<td>displacement vector</td>
</tr>
<tr>
<td>$x$</td>
<td>displacement frequency component</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>rotor speed</td>
</tr>
<tr>
<td>$\omega$</td>
<td>fundamental frequency of steady state solution</td>
</tr>
<tr>
<td>$\omega_k$</td>
<td>frequency of $k$’th harmonic = $k\omega$</td>
</tr>
</tbody>
</table>

SUPERSCRIPTS (IF NOT OTHERWISE DEFINED)
- $t$ differentiation with respect to time $t$
- $c$ connection degree of freedom
- $f$ free (non-connection) degree of freedom

SUBSCRIPTS (IF NOT OTHERWISE DEFINED)
- $b$ bearing
- $e$ external
- $i$ bearing station number: $i = 1, \ldots, 4$
- $k$ order of harmonic
- $p$ pedestal
- $r$ rotor
- $u$ unbalance
- $y, z$ vertical and horizontal directions respectively
3 THEORY

The schematic for a general rotor bearing foundation system with \( m \) hydrodynamic bearings is shown in Fig. 1. The equations of motion for an axially symmetric rotor at some selected constant speed \( \Omega \) are:

\[
M_r \ddot{X}_r + C_r \dot{X}_r + K_r X_r = F_b + F_e + F_u
\]

where the bearing forces \( F_b \) are, in general, non-linear functions of the journal motions, the other external forces \( F_e \) are known (e.g. unidirectional gravity forces), and the unknown unbalance forces \( F_u \) are synchronous with the rotor speed.

![Figure 1: Schematic of a general rotor-bearing-pedestal system.](image)

Assuming periodic steady state solution with fundamental frequency \( \omega \), the solution to eqn (1) may be written as:

\[
X_r = \sum_{k=0}^{n} x_{rk} e^{j\omega_k t}
\]

On substituting into eqn (1) one obtains \((n+1)\) sets of simultaneous equations for the \( x_{rk} \) of the form:

\[
\left[ -\omega^2 M_r + j\omega C_r + K_r \right] x_{rk} = f_{bk} + f_{ek} + f_{uk}
\]

where

\[
f_{ek} = \begin{cases} F_e & \text{when } k = 0 \\ 0 & \text{otherwise} \end{cases}
\]

and

\[
f_{uk} = \begin{cases} F_u & \text{when } k = \Omega/\omega \\ 0 & \text{otherwise} \end{cases}
\]

Similarly, the equations of motion of the foundation are given by:

\[
M_p \ddot{X}_p + C_p \dot{X}_p + K_p X_p = F_p = -F_b
\]

yielding, upon reaching steady state, the following \((n+1)\) sets of simultaneous equations for the \( x_{pk} \):

\[
\left[ -\omega^2 M_p + j\omega C_p + K_p \right] x_{pk} = -f_{bk}
\]

Of prime interest here are the equation sets for \( k = 0 \), for which eqn (3) simplifies to:

\[
K_r x_{ro} = F_e + f_{bo}
\]

As shown in ref. [4], from measurements of the displacement of the journal inside the bearing over a cycle (at each bearing station) it is possible to determine the zero frequency components of the bearing forces \( f_{bo} \). Thus, all the terms on the right hand side of eqn (8) are known, being either given or calculatable from measurements. The deformed rotor centreline, defined by \( x_{ro} \), can then be found with respect to some arbitrary fixed datum line. A convenient datum line (that chosen here), is the line passing through the journal centres at the first and last bearing stations, whereupon the \( x_{ro} \) at these two bearing stations will be zero. With this reduction by four of the number of unknown \( x_{ro} \) values, eqn (8) can be solved to obtain the rotor centreline shape. This is illustrated in Fig. 2 for a four bearing system. Also, from the measured zero frequency eccentricities \( e_i \) of the journals inside the bearings, one can determine the location of the bearing centres relative to the fixed datum line (also shown in Fig. 2).

One next needs to locate the zero frequency location of the bearing housings. For simplicity of exposition, and without loss of generality, it will be assumed that the flexible foundation comprises flexible housing/casing pedestal supports at each bearing station, fixed to a rigid base. The equation sets for \( k = \Omega/\omega \) can then be used to identify the dynamic foundation parameters \( M_p, C_p \) and \( K_p \) without prior knowledge of the system configuration state or system unbalance state [5]. Hence, these parameters may be assumed known.

Now for \( k = 0 \), eqn (7) in partitioned form gives:

\[
\begin{bmatrix} P & Q \\ R & S \end{bmatrix} \begin{bmatrix} x_{po} \\ x_{po}^* \end{bmatrix} = \begin{bmatrix} O \\ -f_{bo} \end{bmatrix}
\]

![Figure 2: Relative location of datum line for 4 bearing system](image)
On eliminating the non-connection displacements \( x_{p0} \), one obtains:

\[
[S - RP^{-1}Q]x_{p}^c = K_p x_{p0}^c = F_{bo} \tag{10}
\]

If each pedestal is identified as having just two degrees of freedom with respect to the fixed rigid base, viz. motions in the lateral \( y \) and \( z \) directions, then \( K_p^c \) will be diagonal. Since \( f_{bo} \) is already known, eqn (10) yields the zero frequency displacements of the bearing housings \( b_t \) from their original unloaded locations. Selecting now a second datum line as that which passes through the centres of the unloaded first and last bearing, the locations of the other unloaded bearing centres relative to this second datum line can be determined (\( a_2 \) and \( a_3 \) in Fig. 2). These relative unloaded bearing centre locations define the configuration state of the system. Note that the system configuration state, and hence this second datum line, is speed independent, being defined by the physical installation of the rotor bearing foundation system, whereas the location of the first datum line relative to this second datum line does change with speed.

4 Numerical Experiments

The capability of this proposed configuration state identification procedure was evaluated by numerical experiments. Figure 3 shows the 8 segment lumped mass representation of the test rotor bearing foundation model. It consists of a horizontal statically indeterminate unbalanced rotor flexibly supported on four equivalent simple circular bore journal bearings at nodes 2, 4, 6 and 8.

![Lumped mass representation of rotor model](image)

**Figure 3: Lumped mass representation of rotor model**

Table 1 lists the relevant input data. The system responses at the three speeds 150, 200 and 220 rad/s were generated by in-house transient analysis software using 4th order fixed time step Runge Kutta integration; and necessitated at least 8000, but occasionally 64,000 time steps per cycle to ensure solution accuracy. Typical steady state response orbits are shown in Fig. 4 for 150 rad/s. Once periodic, steady state conditions were reached the journal orbit eccentricities were sampled to 5 significant digits (128 points per cycle), and constituted the ‘measurements’ for the numerical experiment. From this data, the zero frequency journal eccentricities \( e_1 \ldots e_4 \) and zero frequency bearing forces \( f_{bo} \) were evaluated, the latter values assuming the short bearing approximation with \( \pi \) film (a more exact evaluation of the Reynolds equation, appropriate to the actual bearing design, could be used in practice but was felt unnecessary for the purposes of this paper), using differentiation in the frequency domain to obtain the instantaneous journal velocity [4]. The deflected rotor shape was then obtained from eqn (8), and the zero frequency displacement vectors \( b_1 \ldots b_4 \) of the bearing housings were obtained from eqn (10) enabling the configuration state vectors \( a_2 \) and \( a_3 \) to be determined.

![Steady state response orbits at 150 rad/s](image)

**Figure 4: Steady state response orbits at 150 rad/s**

With 5 digit measurement accuracy, the actual configuration state was recovered exactly (to the nearest \( 10^{-5} \) metres), proving the validity of the technique. However, 5 digit measurement accuracy is not
practically realisable. Thus, the ‘measured’ data was truncated to 2 digits, an accuracy attainable with present day instrumentation. Table 2 lists the actual and identified configuration coordinates (to the nearest \(10^{-5}\) metres) of bearings 2 and 3, both for the unbalanced system (case 1), and also for a balanced system (case 2). For the unbalanced system, the identification obtained is surprisingly good, with best identification obtained at 200 rad/s and worst at 150 rad/s where the response exhibited most non-linearity (as evidenced by the number of harmonics needed to represent the measured response). Most identification inaccuracy occurred with the horizontal location of the bearing; and one would be tempted to accept the identification predictions at 200 rad/s and 220 rad/s in favour of that for 150 rad/s, owing to their virtual equivalence. Note that an error of \(1 \times 10^{-5}\) metres corresponds to an error in journal eccentricity ratio of 0.11. For the balanced system, the 2 digit accuracy identification results are still good but surprisingly, they are worse than for the unbalanced case (which necessitates consideration of journal velocities as well as displacements). This is probably due to the averaging effect in determining the zero frequency journal eccentricity when using results from an orbit. In this latter case, the results for 150 rad/s and 220 rad/s are sufficiently similar to accept their average, yielding an identification prediction for the coordinates of bearings 2 and 3 of (+2, -148) and (-8, -163.5) \(\times 10^{-5}\)m respectively.

**Table 2: Actual and identified configuration states for balanced and unbalanced rotors at various rotor speeds (Bearing coordinates are in units of \(10^{-5}\) m)**

<table>
<thead>
<tr>
<th>Bearing</th>
<th>Actual x</th>
<th>y</th>
<th>150 rad/s x</th>
<th>y</th>
<th>200 rad/s x</th>
<th>y</th>
<th>220 rad/s x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0</td>
<td>-145</td>
<td>-7</td>
<td>-144</td>
<td>-0</td>
<td>-145</td>
<td>-0</td>
<td>-144</td>
</tr>
<tr>
<td>3</td>
<td>-9</td>
<td>-160</td>
<td>-16</td>
<td>-159</td>
<td>-9</td>
<td>-160</td>
<td>-10</td>
<td>-159</td>
</tr>
</tbody>
</table>

**CASE 1: Unbalanced**

**CASE 2: Balanced**

5 CONCLUSIONS

1. A configuration state identification procedure for statically indeterminate unbalanced rotor bearing foundation systems which utilises a knowledge of the displacement of the journals relative to the bearings at some select speed (presuming periodic steady state response has been attained) has been outlined and verified by numerical experiments on a four bearing system.

2. The technique presupposes a sufficiently accurate model of the rotor and the foundation but does not rely on a prior knowledge of the unbalance state.

3. The technique relies on the ability to accurately evaluate the hydrodynamic bearing forces utilising a knowledge of the motion of the journals relative to the bearings. This reliance of the predictive capacity of Reynolds equation is felt to be a major potential weakness.

4. Numerical experiments on a four bearing system have shown that though the accuracy of the technique is sensitive to measurement errors, satisfactory configuration state identification is possible even if journal displacement measurements are accurate to only two significant digits.

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7 REFERENCES


