THE INFLUENCE OF SURFACE ROUGHNESS ON THERMOHYDRODYNAMIC ANALYSIS

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SUMMARY
An approach has been developed to investigate the influence of surface roughness on thermohydrodynamic analysis with film conditions which systemically occur in journal bearings. A parametric investigation is performed for predicting the bearing behaviours such as pressure and temperature distributions in lubricating films between the stationary and moving surfaces determined by absorbed layers and their interfaces on the statistical method for rough surfaces with Gaussian distribution. The layers expressing the effects of surface roughness are expressed as functions of the standard deviations \((\sigma)\) of each surface and surface orientation \((j)\) to explain the flow patterns between both rough surfaces. The coupled effect of surface roughness and shear zone dependency on hydrodynamic pressure and temperature has been found in non-contact mode and contact mode, respectively.

Keywords: Thermohydrodynamic (THD), Journal Bearing, Lubrication, Surface Roughness, Gaussian Distribution

1 INTRODUCTION
Modelling of lubrication, acting under very thin film conditions is generally complicated due to fluidic lubrication, and partial asperity contact. The study of surface roughness effects in micro machine design has intensively been the subject of investigations in recent years. Starting with the work of Cheng, many mathematical models have been suggested as forms which resulted in the averaged Reynolds equation from mean flow quantities [1-4]. As lubricating film thickness decreases to an order of magnitude of rough surface the effect of roughness is most important in the flow patterns. In this regime the system behaviour intensively relates to the manner in which flow patterns undergo severe shear in the film lubrication, heat generation occurs in the shearing liquid, and the neighbouring surface roughness affects performance. In this study, a model is considered for parametric investigation of the influence of the surface roughness on thermohydrodynamic analysis. A parametric investigation accounts for the effects of roughness distribution and the system behaviours in flow condition with the stationary and moving surfaces classified on the statistical tool to explain irregular rough surface patterns.

2 MODEL
2.1 Model Description
A schematic diagram of narrowed zone of two surfaces with a topography in the journal bearing is shown in Fig. 1. For the statistical approach, the amplitudes of the surface roughness \(\delta_i\) \((i = 1, 2)\) are described by the Gaussian distribution with standard deviations \(\sigma_1\) and \(\sigma_2\), respectively. The film profile within a hydrodynamic zone may be represented as follows;

\[
h_t(x, y) = \hat{h} + \delta_1(x, y) + \delta_2(x, y)
\]

where \(\hat{h} = C(1 + \epsilon \cos \theta)\) represents the nominal film thickness between the two smooth surfaces. The film thickness including the average roughness amplitude is then defined as:

\[
h = \hat{h} + 2\left[\int_{0}^{\infty} \delta \psi(\delta) d\delta_1 + \int_{0}^{\infty} \delta \psi(\delta) d\delta_2\right]
\]

(2)

where \(\psi(\delta)\) is the probability density function of \(\delta\). In setting an equivalent flow model for THD analysis, the lubricant is carried within the valleys of the moving rough surface as uniform moving fluid zone with same velocity distribution. On the other hand, the fluid tends to stagnate in the valleys of the stationary rough surface to have zero velocity as a stationary fluid zone. It also makes the hydrodynamic fluid forces to be transported in the bearing gap as shear zone out of both effects. If film gap \(h\) decreases to the order of magnitude of surface roughness parameters \((\sigma_1, \sigma_2)\), surface roughness influences the flow patterns by forming the layers close to surfaces, so that the fluid or solid friction between them significantly affects bearing performance through these layers. At the low ratio, \(\delta\sigma\), a shear zone where the lubricant undergoes a high shear rate showing the effects of surface roughness. It can be determined from the statistical tool where a probability density of asperities does not exceed a critical value of the roughness deviations. Also, it statistically defines the interfacial boundary as follows;

\[
\begin{align*}
SL & \in \text{prob}(h_2 < z < h_2) : \text{stationary - layer} \\
ML & \in \text{prob}(h_2 < z < h_1) : \text{moving - layer} \\
SZ & \in \text{prob}(h_1 < z < h_2) : \text{shear - zone}
\end{align*}
\]

where \(\sigma_1 = \int \delta_1 \psi(\delta) d\delta_1\) and \(\sigma_2 = \int \delta_2 \psi(\delta) d\delta_2\).

For the detailed properties of the asperity at random, the orientation of striation can be expressed in two random variables, \(X\) and \(Y\), by an angle, \(\theta\), between the striated direction and the flow motion direction with coefficient of correlation, \(\nu\), and the ratio of the mean wave lengths, \(\lambda_x\) and \(\lambda_y\), in the \(x\) and \(y\) directions.
\[
\alpha = \left( \frac{A_s}{A_y} \right) \cot \theta \Rightarrow \begin{cases} j_i = \infty & \text{longitudinal} \\ j_i = 0 & \text{transverse} \\ j_i = 1 & \text{isotropic} \end{cases} \quad (4)
\]

where \( \theta = \frac{1}{2} \tan \left[ \frac{\sqrt{(2u_1 \sigma_X \sigma_Y)}}{\sigma_Y - \sigma_X} \right] \). These factors similarly give minor changes in determining their interacting boundaries by asperity distribution in one random variable as follows;

\[
\begin{align*}
SL \in \text{prob} & \left[ -\sigma_1 \left[ 1 + \tanh \left( j_1 - 1 \right) \right] - h/2 < z < 0 \right] \\
ML \in \text{prob} & \left[ 0 < z < \sigma_1 \left[ 1 + \tanh \left( j_1 - 1 \right) \right] - h/2 \right] \\
SZ \in \text{prob} & \left[ 0 < z < \sigma_1 \left[ 1 + \tanh \left( j_1 - 1 \right) \right] - h/2 \right]
\end{align*}
\]

\[
\{ \begin{array}{l}
\theta = \pi/4 \leftrightarrow \sigma_X = \sigma_Y \\
\theta = 0 \leftrightarrow \sigma_X > \sigma_Y \\
\theta = \pi/2 \leftrightarrow \sigma_X < \sigma_Y
\end{array} \} \quad (5)
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\[
\begin{align*}
&= \int_{j_1}^{\infty} \left( \delta_J \right)^2 dJ, \\
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\end{align*}
\]

\[
A = \frac{\frac{z}{\mu} - G_0}{G_0} \frac{1}{\mu} dz, \quad B = \frac{\frac{z}{\mu} - G_0}{G_0} \frac{1}{\mu} dz
\]

The following boundary conditions of shear zone are applied:

\[
u_1 = U, \quad u_2 = 0, \quad v_2 = \frac{\partial u_1}{\partial z_1} = \frac{\partial u_2}{\partial z_2} = \frac{\partial v_1}{\partial z_1} = \frac{\partial v_2}{\partial z_2} = 0
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\end{array} \} \quad (5)
\]
asperity contact on corresponding surface area $dA$, all the heat is generated in the shear zone inside the fluid film controlled by the surface roughness as well as in the interfacial boundary zone controlled by the interacting asperities surrounding the average gap.

At the fluid-fluid interface, the temperature B.C.'s associated with the energy equation for the shear zone can simply be considered as; \[ T(0, y, z) = T_{0}, \]
\[
\frac{\partial T}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial T}{\partial y} = 0
\]

For the constant velocity layer, the B.C.'s are,
\[
\frac{\partial T}{\partial z} \bigg|_{z=h} = \frac{\partial T}{\partial z} \bigg|_{z=h} = \frac{\partial T}{\partial z} \bigg|_{z=h} = 0
\]

The viscosity is assumed to follow the exponential relationship given by;
\[
\mu = \mu_0 e^{-\gamma(T - T_0)}
\]

The modified Reynolds equation for taking into consideration the roughness surface is set into finite difference form using central difference scheme. The final form is reproduced here.

\[
\Phi_i \mathbf{P}_{i,j} + \Phi_0 \mathbf{P}_{i-1,j} + \Phi_1 \mathbf{P}_{i+1,j} + \Phi_2 \mathbf{P}_{i,j+1} + \Phi_3 \mathbf{P}_{i,j} = \Phi_4
\]

where $\Phi_i$'s $(i = 1, 2, 3)$ are coefficients applied at the element. The elliptic partial differential form is solved using the SOR scheme. Energy equation is of parabolic type and the ADI is used to obtain the finite difference scheme.

\[
\bar{T}_i = A_0 + A_1 \bar{T}_i + A_2 \bar{T}_i + A_3 \bar{T}_i + A_0
\]

where $A_i$'s $(i = 1, 2, 3)$ are linear operators. The film is discretized into $100x20x60$ not equi-sized meshes.

### 4 RESULTS AND DISCUSSION

The design and test parameters of the considered systems were selected from previous works and are given in Table 1. The computational procedure was used to iteratively evaluate the bearing performance in non-contact mode and contact mode, respectively.

Table 1: Journal bearing data for [4-6]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal radius</td>
<td>$R$</td>
<td>36</td>
<td>$10^{-3}$ m</td>
</tr>
<tr>
<td>Bearing length</td>
<td>$L$</td>
<td>10, 21, 42, 72</td>
<td>$10^{-3}$ m</td>
</tr>
<tr>
<td>Radial bearing clearance</td>
<td>$C$</td>
<td>0.100</td>
<td>$10^{-3}$ m</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>$\varepsilon$</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Lubricant viscosity</td>
<td>$\mu$</td>
<td>0.0277</td>
<td>Pa-s</td>
</tr>
<tr>
<td>Lub. thermal conductivity</td>
<td>$K$</td>
<td>0.13</td>
<td>W/m-K</td>
</tr>
<tr>
<td>Lubricant density</td>
<td>$\rho$</td>
<td>860</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Thermo- viscosity coeff.</td>
<td>$\gamma$</td>
<td>0.0298</td>
<td>1/C</td>
</tr>
<tr>
<td>Specific heat</td>
<td>$c_p$</td>
<td>2000</td>
<td>J/kg-K</td>
</tr>
<tr>
<td>Shaft speed</td>
<td>$N$</td>
<td>500–2000</td>
<td>rpm</td>
</tr>
<tr>
<td>Young modulus</td>
<td>$E$</td>
<td>2.1</td>
<td>$10^{11}$ Pa</td>
</tr>
<tr>
<td>Bulk modulus</td>
<td>$\beta$</td>
<td>1.517</td>
<td>$10^{-3}$ N/m$^2$</td>
</tr>
<tr>
<td>Thermal expansion</td>
<td>$\alpha$</td>
<td>0.019</td>
<td>m/m-K</td>
</tr>
</tbody>
</table>

Figure 3: Probability density surfaces for contact or near contacting surfaces

Figure 4: Effect of surface roughness on the maximum pressure with different $L/D$ ratios, $R=36 \times 10^{-3}$ m, $C=100 \times 10^{-3}$ m, $\varepsilon=0.9$, $T_0=40^\circ C$, $N=1000$ RPM

Figure 5: Effect of surface roughness on the maximum temperature with different $L/D$ ratios, $R=36 \times 10^{-3}$ m, $C=100 \times 10^{-3}$ m, $\varepsilon=0.9$, $T_0=40^\circ C$, $N=1000$ RPM

The observations displayed from the Figs 4 and 5 show the effect of surface roughness parameter $h/\sigma$, for aspect ratio of 0.13 to 1.0. It may be seen that when the film thickness is of the same magnitude as the roughness height, the effect of surface roughness is significant on bearing characteristics. Especially, with the film...
thickness close to \( h/\sigma \sim 1 \) or higher L/D aspect ratio, the bearing characteristics in the pressure changes had significant increases from the smooth bearing solution. Similarly, the same trend occurred in the temperature changes. It is noted that roughness increases the peak values of the pressure and temperature.

![Graph showing the effect of speed on the maximum pressure](image)

**Figure 7:** Average contact pressure for various surface roughness parameters along the centreline in the narrow zone of the journal bearing, \( R = 36 \times 10^{-5} \text{ m} \), \( L = 21 \times 10^{-3} \text{ m} , C = 100 \times 10^{-6} \text{ m} , \epsilon = 0.9 , T_0 = 40^\circ \text{C} , N = 1000 \text{ RPM} \)

Fig. 6-(a) illustrates the effect of speed on the maximum pressure. It is noted that pressure distribution along the circumferential direction for L/D=0.29 and below \( h/\sigma = 1.5 \) is shown Fig. 6-(b) for surface roughness parameter, \( h/\sigma \). The corresponding temperature distribution across the film is shown in Fig. 6-(c). It can be observed that the temperature distribution significantly changes after the high pressure-region in the shear zone.

### 4.2 Contact mode, \( \sum d_i > 0 \)

When frictional heat is released along the central contacting interface on the differential area, the effect of surface roughness is much significant on bearing characteristics than the one of non-contact mode. The predictions are indirectly explained in Fig. 7. Here, values \( n r \sigma \sim 0.05 \) and \( \sigma / r \sim 0.029 \) are used to give the average pressure according to penetration depth on the tip asperity with varying \( h/\sigma \) in the contact mode. As the penetration depth \( (d/\sigma) \) is increased 0.01 to 0.3 at L/D=0.29 the average pressure logarithmically increased and slightly showed differences with the minor changes in \( h/\sigma \). It is noted that the increasing roughness parameter, \( h/\sigma \), in contact mode approaches to over 3\( \sigma \) for decreasing or vanishing contact areas.

![Graph showing average contact pressure](image)

**Figure 6 (a) Journal bearing pressure speed characteristics, (b) Pressure distribution for various surface roughness parameters along the centreline of the journal bearing in the direction of sliding motion, (c) Dimensionless temperature distribution at the mid plane in the sliding direction, \( R = 36 \times 10^{-5} \text{ m} , L = 21 \times 10^{-3} \text{ m} , C = 100 \times 10^{-6} \text{ m} , \epsilon = 0.9 , T_0 = 40^\circ \text{C} , N = 1000-2000 \text{ RPM} \)**

### 5 CONCLUSIONS

A new approach is based on defining the shear zone in terms of parameters \((\sigma, j)\) which are functions of surface roughness characteristics. The model is designated to suit fluid flow field that is stagnated and has slipped to rough surfaces. The behaviour of a laminar Newtonian in a finite width rough journal bearing is investigated. The analysis reveals that: 1. The present generalized Reynolds equation is able to incorporate the coupled modified velocity and energy equations for each flow pattern, 2. As the roughness parameters are increases the load carrying capacity becomes higher, and 3. Combined effect of roughness and their contact behaviour between surfaces is vanished by returning Newton fluid behaviour in thin film as approaching the hydrodynamic regime.

### 6 REFERENCES


