THE PRESSURE DISTRIBUTION IN A HERTZ LINE CONTACT AFFECTED BY A LONGITUDINAL FURROW

E.N. DIACONESCU and D.F. CERLINCA
University of Suceava, 1 University Street, 5800 Suceava, ROMANIA, e-mail: emdi@usv.ro, deliacerlinca@yahoo.com

SUMMARY
This paper intends to improve the work of Chiu and Liu and to provide an analytical solution to the problem of a line Hertz contact in the presence of a longitudinal furrow. To this end, the contact is modelled by an equivalent perfect rigid cylinder having an axial furrow pressed against an equivalent elastic half-space bounded by a smooth plane. Starting from the work of Muskhelishvili, the solution is found analytically for the furrow being placed either centric or eccentric with respect to the contact centre. Because the solution is complicated, the results are found by evaluating numerically the pressure equations. The effects of several geometric parameters of the furrow are evidenced.

Keywords: surface defects, contact pressure, Hertz line contact, furrow

1 INTRODUCTION
Rolling contact fatigue, as one of most encountered failures in machine details is strongly influenced either by surface defects (dents, furrows or large asperities) or by subsurface imperfections (voids, inclusions, oxides or microcraks). As the quality of bearing steel improved significantly during the last decades, the surface defects tend to become the major factor influencing contact fatigue.

Aware of the importance of surface quality, Chiu and Liu, [1], attempted to investigate the stress riser effect around a furrow type defect present in a Hertz line contact. By assuming that the contact width is extremely large in comparison to furrow dimensions, they modelled the problem by loading with uniform pressure an elastic half-space that possesses an infinitely long furrow on the plane boundary. The furrow has a constant cross-section shown in Figure 1 and therefore a plane problem occurs. The pressure found by a complex potential method advanced by Muskhelishvili, [2], presents important stress risers on the shoulders of the furrow, tends asymptotically towards a constant value at infinity and vanishes inside the furrow.

Gao, Dwyer-Joyce and Beynon, [3], report numerical results obtained by matrix inversion method in the case of a 2-D contact in the presence of a furrow of specified geometry. They observed very high pressure at the dent shoulders that may cause plastic flow.

This paper aims to perform a theoretical analysis of the problem in order to assess simply the pressure riser effects that occur when the furrow takes various positions with respect to contact centre and the furrow geometry may change.

2 PRESSURE DISTRIBUTION
In order to evaluate the contact pressure between two parallel cylinders, one bounded by a smooth surface and the other possessing an axial furrow, the contact is modelled as follows. An equivalent perfect rigid cylinder possessing the axial furrow is pressed against an equivalent elastic half-space bounded by a smooth plane. The plane of symmetry of the furrow can be positioned in various position with respect to the initial contact, either centric or eccentric. When it is placed in the central plane of the contact, the problem is symmetrical, as in Figure 2a and it becomes eccentric for any other position, as shown in Figure 2b.

A schematic view of the asymmetric contact used in the calculations below is presented in Figure 3.
Figure 3: Geometry of asymmetric placed furrow

The equation of the punch surface can be written as follows:

\[
z(x) = \frac{x^2}{2R} + \frac{(c - |x - d|)}{2r} \quad \text{for} \quad \left\{ \begin{array}{l} d - c < x \leq d - b_1; \\ d + b_2 < x \leq d + c; \\ \end{array} \right. 
\]

\[
z(x) = \frac{x^2}{2R} \quad \text{for} \quad x \geq d-c. 
\]

By adequately applying the solution of Muskhelishvili, as improved by Hills, Novell and Sackfield, [4], the pressure distribution can be written in an integral form as follows:

\[
p(x) = 4\mu \cdot \sqrt{B^2 - x^2} \left[ \frac{\int_{d-b_1}^{d-c} z'(t)(t-x) \sqrt{B^2 - t^2} \, dt}{-b} + \frac{\int_{d+1}^{d+b_2} z'(t)(t-x) \sqrt{B^2 - t^2} \, dt}{d+1} \right], 
\]

where \( z' \) is the derivative of \( z \) from equation (1) and has the following expressions:

\[
z'(x) = \frac{x - c - x + d}{r} \quad \text{for} \quad d + b_2 \leq x < d + c; 
\]

\[
z'(x) = \frac{x - c - d + c}{r} \quad \text{for} \quad d - c < x \leq d - b_1; 
\]

\[
z'(x) = \frac{x}{R} \quad \text{for} \quad x \leq d - c \quad \text{and} \quad x \geq d + c. 
\]

In equation (2) the third integral is zero because in this region there is no contact. Therefore, the pressure distribution can be written as follows:

\[
p(x) = 4\mu \cdot \sqrt{B^2 - x^2} \left[ I_1(x) + I_2(x) + I_3(x) + I_4(x) \right], 
\]

where \( I_j \) are the symbols for following integrals:

\[
I_1(x) = \frac{1}{R} \int_{-b}^{d+b_2} \frac{z'(t)(t-x)}{(t-x) \cdot \sqrt{B^2 - t^2}} \, dt; 
\]

\[
I_2(x) = \frac{1}{r} \int_{-b}^{d+b_2} \frac{z'(t)(t-x) \cdot \left(1 + \frac{x}{R}\right)}{(t-x) \cdot \sqrt{B^2 - t^2}} \, dt; 
\]

\[
I_3(x) = \frac{1}{R} \int_{d-b_1}^{d+b_2} \frac{z'(t)(t-x)}{(t-x) \cdot \sqrt{B^2 - t^2}} \, dt; 
\]

\[
I_4(x) = \frac{1}{R} \int_{d-b_1}^{d+b_2} \frac{z'(t)(t-x) \cdot \left(1 + \frac{x}{R}\right)}{(t-x) \cdot \sqrt{B^2 - t^2}} \, dt. 
\]

The substitution of equations (3) into equations (5) leads to:

\[
I_1(x) = \frac{1}{R} \int_{-b}^{d+b_2} \frac{z'(t)(t-x)}{(t-x) \cdot \sqrt{B^2 - t^2}} \, dt; 
\]

\[
I_2(x) = \frac{1}{r} \int_{-b}^{d+b_2} \frac{z'(t)(t-x) \cdot \left(1 + \frac{x}{R}\right)}{(t-x) \cdot \sqrt{B^2 - t^2}} \, dt; 
\]

\[
I_3(x) = \frac{1}{R} \int_{d-b_1}^{d+b_2} \frac{z'(t)(t-x)}{(t-x) \cdot \sqrt{B^2 - t^2}} \, dt; 
\]

\[
I_4(x) = \frac{1}{R} \int_{d-b_1}^{d+b_2} \frac{z'(t)(t-x) \cdot \left(1 + \frac{x}{R}\right)}{(t-x) \cdot \sqrt{B^2 - t^2}} \, dt. 
\]

The definite integrals (6) have quite extensive analytical expressions that are shown in Appendix 1. After some routine calculation, the expression for general pressure distribution can be written in the following form:

\[
p(x) = \frac{1}{\pi} p_0 \cdot I(x), 
\]

where \( p_0 \) is maximum Hertz pressure when the furrow is absent, and \( I(x) = I_1(x) + I_2(x) + I_3(x) + I_4(x) \).

In order to speed up the calculations, the values of integrals \( I_j \) and consequently of pressure are evaluated numerically by using an adequate soft like the Mathcad. Typical results for pressure distribution obtained for different dimensionless geometrical parameters involved in the analysis are shown in Figures 4 to 9. Dimensionless values are obtained by dividing the actual lengths by \( b \) and the pressure by \( p_0 \).

The effect of furrow position is illustrated in Figure 4.
A synthesis of numerical results concerning the effect of furrow position upon pressure distribution is shown in Figure 5. A stress riser $k_p$ is defined as the ratio between recorded maximum pressure and $p_0$.

The radius $r$ can influence substantially the pressure riser. In order to evaluate its influence, various radii were chosen and, preserving constant the other geometrical parameters, the pressure distributions were evaluated. Typical results are illustrated in Figure 6.

As expected, the diminution of radius $r$ leads to an increase of pressure riser. This behaviour is better evidenced in Figure 7 which synthesises the results of numerical analysis.

As mentioned, the contact area extends towards the centre of the furrow leaving a gap $2b$ or $b_1+b_2$. The effect of this gap upon pressure riser is shown as examples in Figure 8.
These three figures indicate an increase of pressure riser as the gap \( b \) reduces. The numerical results obtained are concentrated in Figure 9.

Figure 9: General effect of gap \( b_1 + b_2 \) upon pressure riser

3 CONCLUSIONS

A new analytical method was derived to assess the pressure distribution in a line Hertz contact when one of the contacting cylinders possesses a longitudinal furrow. The results obtained yield the following observations.

The presence of a furrow modifies the Hertz pressure only in its immediate neighbourhood, producing significant stress risers on the shoulders and a dip in its central region. Away of furrow, the pressure remains essentially hertzian, although the global width of the contact strip increases slightly. These effects are obvious in both cases, symmetric and asymmetric.

The pressure riser depends both on the geometry of the furrow and on its position with respect to the initial contact line. Numerical results show that maximum riser effect occurs when the furrow is eccentric by a critical position that is affected by contact geometry.

The effect of pressure rising increases as the radius \( r \) and the gap \( 2b \) decrease.

4 REFERENCES


APPENDIX 1

Analytical expressions of integrals \( I_i \)

\[
I_1(x) = \left[ \frac{1}{R} \arcsin \left( \frac{d+c}{B} \right) - \frac{1}{R} \arcsin \left( \frac{d-c}{B} \right) - \left( \frac{1}{R} + \frac{1}{R} \right) \arcsin \left( \frac{d+b_2}{B} \right) - \arcsin \left( \frac{d-b_1}{B} \right) \right] \sqrt{B^2-x^2} + \frac{x \cdot \arctanh \left( \frac{\sqrt{B^2-x^2} \cdot \left[ B^2-x \cdot (d-c) \right] \sqrt{B^2-(d-b_1)^2} - B^2-x \cdot (d+b_2) \right) \sqrt{B^2-(d+c)^2}}{B^2-(d-c)^2 \cdot \sqrt{B^2-(d-c)^2} - B^2-x \cdot (d-c) \sqrt{B^2-(d+c)^2}} \right];
\]

\[
I_2(x) = x \left( \frac{1}{R} + \frac{1}{R} \right) \cdot \arctanh \left( \frac{\sqrt{B^2-x^2} \cdot \left[ B^2-x \cdot (d+b_2) \right] \sqrt{B^2-(d+b_2)^2} - B^2-x \cdot (d-b_1) \right) \sqrt{B^2-(d+c)^2}}{B^2-(d+b_2)^2 \cdot \sqrt{B^2-(d+b_2)^2} - B^2-x \cdot (d-b_1) \sqrt{B^2-(d+c)^2}} \right];
\]

\[
I_3(x) = \frac{d-c}{r} \cdot \arctanh \left( \frac{\sqrt{B^2-x^2} \cdot \left[ B^2-x \cdot (d-b_1) \right] \sqrt{B^2-(d-b_1)^2} - B^2-x \cdot (d-c) \right) \sqrt{B^2-(d-c)^2}}{B^2-(d-b_1)^2 \cdot \sqrt{B^2-(d-b_1)^2} - B^2-x \cdot (d-c) \sqrt{B^2-(d-c)^2}} \right];
\]

\[
I_4(x) = \frac{d+c}{r} \cdot \arctanh \left( \frac{\sqrt{B^2-x^2} \cdot \left[ B^2-x \cdot (d+c) \right] \sqrt{B^2-(d+b_2)^2} - B^2-x \cdot (d-b_1) \right) \sqrt{B^2-(d+c)^2}}{B^2-(d+b_2)^2 \cdot \sqrt{B^2-(d+b_2)^2} - B^2-x \cdot (d+c) \sqrt{B^2-(d+c)^2}} \right].
\]