AN ASPERITY BASED MODEL FOR FRICTION IN MIXED LUBRICATION

F. ROBBE-VALLOIRE, B. PAFFONI, R. PROGRI, R. GRAS
ISMCM-CESTI, LISMA Tricologie, 3, rue Fernand Hainaut, 93407 Saint-Ouen CEDEX, FRANCE;
e-mail: francois.robbie-valloire@luxor.ismcm-cesti.fr

SUMMARY
We propose a model for mixed lubrication between parallel surfaces. To do such, we first analyze the microgeometry. We determine a statistical distribution for asperity height and radius. Drawing upon these geometric parameters we identify the main mechanism available on a given asperity. Wherefrom we deduce the individual normal and friction forces. The five possible mechanisms are iso- or piezo-viscous hydrodynamic, elastohydrodynamic, elastoplastic and full plastic. By association of the results obtained for each asperity, we derive the mechanical behaviour for the entire contact. Several numerical applications are proposed. Theoretical results show the influence of the main contact parameters. They strongly agree with experimental results and theoretically confirm the master curve obtained by plotting the evolution of the friction coefficient versus the $\eta V/\rho$ parameter with respect to a given pair of materials and their microgeometry.

Keywords: Asperity, Microgeometry, Coefficient of friction, Mixed lubrication

1 INTRODUCTION
Mechanical engineering often uses fluid lubrication for contacts. In the majority of cases, a continuous film separates the two parts in contact, thus lowering friction and reducing wear consequently lengthening service life. The thickness of the fluid film between the two surfaces depends on the working conditions of the contact. An increase in the normal load or a decrease in the speed can reduce the film thickness. In particular cases such as at start-up or overloading, the film thickness can be so thin that contact arises at the summit of asperities, thus increasing the friction force and wear. This situation - the so-called mixed or limit lubrication, is suitable in mechanisms such as clutches, synchromeshes or brakes because the level of friction arising is stable over time and exhibits no abrupt shifts during operation. In both situations - contact in either transient or continuous work - it is necessary to know the instantaneous level of the friction force and its evolution with changes in service conditions.

This research involves a contact geometry similar to that observed in situations such as those found in clutches or brakes. The surfaces are parallel. We propose a theoretical analysis of the friction force variation with the contact parameters (normal load, sliding speed, viscosity, microgeometry and mechanical characteristics of the two materials in contact). Microgeometry is described with standardised roughness and waviness parameters. The mechanical characteristics used are Young’s modulus, Poisson’s ratio and yield stress.

2 STATE OF THE ART
Experiment after experiment have clearly demonstrated that a large load carrying capability appears even if surfaces are parallel. As a classical result (Figure 1) the friction coefficient seems to be dependant on the parameter $\eta V/\rho$.

It is difficult to give a complete bibliography of experimental results because of the imposing number of publications. For example, it is interesting to examine experimental results on face-sealing by Lebeck [1], on deep-drawing by Emmens [2] or Felder [3].

![Figure 1: Strubeck curve](image)

The two latter authors, according to the range of materials tested, suggest new parameters associations which include the roughness of surfaces. Emmens proposes $H_e = \eta V/\rho R_p^2$ and adds the $R_p$ (altitude of the mean line of the profile from the lowest point). Felder proposes the adimensional parameter $H_f = 3\eta V/\rho R_a^2$ where $R_a$ is the arithmetic mean of the height of the profile and $l$ describes the space between two consecutive asperities.

On the basis of their experiments, the use of these parameters gives a better localisation of the points near a single curve (or master curve) than the use of the classical parameter $\eta V/\rho$.

Analysis of the theoretical results reveals a great variety of mechanisms capable of providing a load carrying capability in a lubricated contact with parallel surfaces. Lebeck [4] offers a wide-ranging survey of these mechanisms and analyses deviation from parallel effects such as undesirable pad tilting or thermal deformation,
squeeze effect coupling with dynamic behaviour of the supports. But the most promising way seems to be a microasperity lubrication effect. At the scale of the microgeometry surfaces are not really parallel because the roughness effect and the small variations of fluid thickness give a resulting load carrying capability through the fluid. Solid contacts are present at the summit of the asperities in contact. Thus the normal load is transmitted through direct solid contact, with a large friction coefficient at the summit of the highest asperities and through intermediate fluid contact, with a low friction coefficient at the asperities situated below the height required for direct contact. The relative proportions of these two types of contact determine the global friction coefficient for the contact.

This analysis agrees very well with experimental results, in particular those of Emmens or Felder, showing roughness parameters as determinant parameters in mixed lubrication. We propose to improve it with a predictive analysis of the share due to forces (normal or tangential) respectively acting with direct or fluid contact.

3 METHODOLOGY OF THE THEORETICAL ANALYSIS

A pertinent analysis of the micro asperity lubrication first needs a complete description of the microgeometry at the scale of the asperity and secondly requires for each asperity the knowledge of its mechanical behaviour. Finally the association of results obtained at each asperity gives the global behaviour of the contact. This paragraph rapidly presents the principle of each of the three steps in the methodology. More details can be found in the next paragraphs.

3.1 Microgeometry-description

The decomposition of the microgeometry into asperities constitutes a common way to analyse the contact between rough surfaces. The classical assumptions for this decomposition often are the following.

- Microgeometry is periodically spaced [5, 6, 7, 8].
- Asperity peaks are approached by a spherical cup and have a constant shape, (constant radius of the asperities summit) [7, 8].
- Distribution of summit follows a normal distribution [7, 8].

Some surfaces agree very well with these assumptions. But it is very usual to find surfaces showing a great variability in summit heights, radii and spacing [9]. Thus we propose to include this variability in the micro asperity lubrication through use of the statistical distribution for height and radius.

3.2 Mechanical behaviour of asperity

Each asperity can act through several way and as an example, let us consider a particular situation of the contact of a rough deformable surface on a smooth rigid one (Figure 2). The general case of two rough surfaces in contact will be detailed in a subsequent paragraph.

As one can see due to the variability of summit heights, the contact includes asperities working under different conditions. Classical analysis proposes a duo family decomposition of the asperities with the first family in direct contact (those having interference) and the rest acting in fluid contact. We propose in the following model a more detailed decomposition based on the classical phenomena observed at the macroscopic scale. When we perform an phenomenological analysis from the asperity furthest from the counterface to the most deeply indented one, it is possible to identify the following mechanisms:

- Isoviscosity and Piezoviscosity Hydrodynamics (PVR) : Due to the fluid film thickness fluid pressure is low and insufficient to cause elastic deformation of the asperity or change in viscosity. Calculation in this case is based on the assumption of rigid bodies with isoviscosity fluid. By decreasing the film thickness, the fluid pressure increases and the initial effect is a change of viscosity. The change in viscosity is describe by the classical exponential Barus's law for piezoviscosity.
- Elastohydrodynamic asperity (EHD) : For lower film thicknesses, the piezoviscosity effect remains but the pressure increase elastically deforms the asperity. In this case we adopt the classical solution for elasto-hydrodynamic lubrication.
- Elastoplastic deformation (EP) : Because of the indentation effect, stress locally can exceed the yield stress of the material. The contact mechanism is the so-called elatoplastic deformation because the existence of both elastic and plastic deformation. The greater the indentation, the higher the ratio of plastic deformation.
- Full plastic deformation (P) : This mechanism occurs when the plastic deformation is generalised in the area of contact.

3.3 Association of these results

The methodology consists in analysing the contact with a given relative position for the distance between the smooth surface and the mean line of the rough antagonist. Afterwards, we quantify the corresponding normal and tangential load by summation of the elementary loads acting on each asperity. By changing the position of the smooth surface, it is possible to analyse every possible normal load on the contact and to obtain the corresponding friction force.

Quantification of the normal and tangential forces on the contact, first consists in checking, over all the contact area, the quantity of asperities having a given height and radius. Secondly, for this kind of asperity, we determine the mechanism. Finally, using the specific law adapted to the identified mechanism, it is possible to obtain the elementary normal and tangential loads for each
asperity. The products of the elementary force by the number of asperities with such geometry give the normal and tangential forces corresponding to all the asperities in the contact having the given altitude and radius. Generalisation of this calculation at all combinations of radii and heights present on the surface gives the total normal and tangential loads for the whole surface.

4 MICROGEOMETRY DESCRIPTION

The microgeometry description is based on a profilometric measurement. A standardised procedure [10] gives for the profile a decomposition of juxtaposed asperities. This procedure uses a four rules algorithm which distinguishes significant peaks and valleys. The representation of this decomposition is proposed in Figure 3 for a face turned surface. Significant peaks are always identified by a vertical line and the deepest valley between two consecutive peaks appears with a horizontal line at the altitude of the valley. In this procedure the portion between two consecutive peaks is termed a motif. This first decomposition gives the roughness motif.

Four standardised parameters characterise the dimensions of the roughness motif:

- $R$ is the mean height of the motif's
- $SR$ is the root mean square of the motif's' heights,
- $AR$ is the mean width of the motif's
- $SAR$ s the root mean square of the motifs' widths.

The standardised procedure contains a second operation which consists in keeping only the peaks. Applying the same procedure with the peaks, it is possible to identify the waviness motif. Analyse of the dimension of these waviness motifs gives 2 other parameters describing the vertical distribution of the peaks:

- $W$ is the mean vertical amplitude between peak,
- $SW$ is the root mean square of peaks amplitude.

For a given surface the statistical distribution for peak altitudes and radii is obtained with a statistical analysis of these standardised parameters. For detailed examination of the full calculation a specific publication [5] should be examined. We propose hereafter the main results obtained.

The proposed distribution quantifies the odds of having one asperity with a given altitude and radius. Distribution with the altitude and the radius are supposed independent.

4.1 Peak height distribution

The analysis of $W$ and $SW$ parameters gives the distribution of peak altitudes. Taking the origin of altitudes at the level of the mean line of the profile, the distribution may be expressed as :

$$f_1(Z) = \frac{1}{SAlt \sqrt{2\pi}} \exp\left( -\frac{1}{2} \left( \frac{Z - Alt}{SAlt} \right)^2 \right)$$

In this formula the mean value of the peak altitude is $Alt = R / 2$ and the standard deviation of the distribution is $SAlt = 0.35 W^2 + SW^2$. As the majority of authors [7, 8], we adopt a normal distribution for altitude of peaks. Distribution of peak altitude on both surfaces studied are illustrated on figure 4.

4.2 Radius distribution

The analysis of the four roughness parameters $R$, $SR$ $AR$ and $SAR$ gives quantitative information of the asperities radius distribution. Adopting a lognormal distribution we obtain the expression :

$$f_2(R^*) = \frac{1}{a \sqrt{2\pi}} \exp\left( -\frac{1}{2} \left( \log R^* - b \right)^2 \right)$$
\[ a = \sqrt{\log \left( \frac{S_{Rad}}{R_{Rad}} \right)^2 + 1} \]
with
\[ b = \log \left( \frac{R}{S_{Rad}} \right) - \log \left( \frac{S_{Rad}^2}{R_{Rad}} + 1 \right) \]

Constants in this formulation are combinations of parameters and adimensional variables \( R^* = \frac{R}{S_{Rad}} \) are suitable:

\[ R_{Rad} = \frac{1}{16} \frac{AR^2 + SAR^2}{R} \]
\[ S_{Rad} = \frac{1}{16} \frac{AR}{R} \left( \frac{SR^2}{R} + 4 \frac{SAR^2}{AR} \right) \]

Radius distribution for both surfaces shown in Figure 5, exhibits large difference between both surfaces and illustrates the importance of the dispersion of the radius for each surface.

![Figure 5](image-url)

Figure 5: Radius distributions for face turned (continuous) and polished (dashed) surfaces

### 4.3 Height and radius distribution on a surface

If we made the assumption of an isotropic surface, characteristics of the surface could be approximated by results deduced from the profile and it would be possible to calculate, on a surface area with a total of \( N_o \) asperities, the number of particular asperities having radii between \( R_1 \) an \( R_2 \) and altitude between \( z_1 \) an \( z_2 \).

\[ N = N_o \int_{z_1}^{z_2} \int_{R_1}^{R_2} f_1(Z) f_2(R) \, dZ \, dR \]

Determination of \( N_o \) is made using the relation proposed by Nayak [6] between the linear density \( dI = 1 / AR \) deduced from profilometry and the surface density \( dS \). The total number of asperities on the whole surface with an area \( A_s \) is \( N_o = 1.2 A_s / AR^2 \).

Table 1 gives the numerical values of roughness and waviness parameters and shows the characteristics of radius and peak distributions (mean and root mean square values).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Face turned</th>
<th>Polished</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roughness motif height (µm)</td>
<td>0.33/0.15</td>
<td>0.125/0.070</td>
</tr>
<tr>
<td>Roughness motif width (µm)</td>
<td>150/60</td>
<td>175/125</td>
</tr>
<tr>
<td>Waviness motif height (µm)</td>
<td>0.17/0.10</td>
<td>0.90/0.035</td>
</tr>
<tr>
<td>Altitude (µm)</td>
<td>0.165/0.110</td>
<td>0.063/0.049</td>
</tr>
<tr>
<td>Radius (mm)</td>
<td>4.9/3.8</td>
<td>22.9/23.1</td>
</tr>
</tbody>
</table>

Table 1: Parameters of microgeometry and statistical distribution on studied surfaces

### 5 NORMAL AND FRICTION FORCES AT ONE ASPERITY

As seen previously, because of the variability in height and radius, the contact between the rough surface and the antagonist can reveal several phenomena such as:

- Iso or piezo viscosity of the fluid,
- Elastic and (or) plastic deformation,
- Direct or fluid contact.

One of the features of the model we propose, takes into account all these phenomena. Unfortunately, this is no single law covering all these aspects. Thus we have try to subdivide into four families the working conditions of the microcontact. For each of these four families, the scope of the domain is detailed (Figure 6). Moreover and various assumptions applied to each domain yield closed expressions for the forces (both normal and tangential) acting on a single asperity.

![Figure 6](image-url)

Figure 6: Description of the limits between succesive mechanisms

The four domains thus obtained are successively detailed and for each of them we express the analytic expression of loads and the corresponding domain of application.

#### 5.1 Iso Piezo Viscous Rigid mechanism (PVR)

The first domain corresponds to a fluid contact situation and is characterised by the highest local values of the film fluid. Thus it corresponds to the furthest asperities from the counterpart.

The assumptions made are the following:

- The solids are in this case rigid,
- The fluid follows a Barus's law.

This domain associates the classical and well known iso piezoviscous hydrodynamic conditions.

To escape the necessity of lengthy numerical calculations we propose analytic approximation laws des-
cibed elsewhere [11]. These laws are expressed for sake of brevity by the following adimensional parameters:

\[ U = \eta_s(u_1 + u_2) / 2 E_p R, \quad H_0 = h_o / R \quad \text{and} \quad G = \alpha E_q \]

Dimensionless normal load \( W_{Fr} \) expresses as:

\[ W_{Fr} = \frac{8.0881}{H_0^{7/2}} \left( 1 - 0.1933 \log \left( 1 - 1.045 G / H_0^{1.5} \right) \right) \]

Friction force \( F_{Fr} \) for a single asperity is:

\[ F_{Fr} = 4U \left( 13.625 - 1.4303 \log(1 - 1.9947 G / H_0^{1.5}) - 0.2498 \log(1 - 0.9558 G / H_0^{1.5}) \right) \]  

Quantitative analysis of the evolution of the normal load shows a rapid increase at a given value \( H_o^{1.5} \). In this case, the assumption of rigid solids becomes unacceptable. Thus we have chosen to stop the use of piezoviscous hydrodynamic near this value. The validity domain for hydrodynamic situation is:

\[ H_o^{1.5} = 1.03007(GU)^{1.5} \]  

5.2 ElastoHydroDynamic mechanism (EHD)

If the adimensional film thickness is lower, elastic deformation becomes significant and the asperity works with the classical assumptions of the elastohydrodynamic regime:

- Deformation in both solids is elastic,
- The fluid obeys the classical piezoviscosity effect described by the Barus' law,
- Fluid film thickness is governed by the Harmrock expression [13],
- Normal load expression is close to that observed in dry situations.

Quantitative analysis of these assumptions is thoroughly described in a specific publication [12] and we propose only the final results which consist in the following approximate law for the normal load \( W_{EHD} \):

\[ W_{EHD} = 2\Delta_{EHD} / 3; \Delta_{EHD} = H_p + a H_o + b H_o^2 / H_p \]  

\( H_p = 1.8058 \left( \frac{E}{G} \right)^{0.55} \left( \frac{U}{\eta} \right)^{0.67}; \quad a = -0.922432 \]

\( b = 0.007365 \)

In addition, it is possible to obtain an expression for the friction force. For this, we consider an isothermal regime in the contact and we limit the shear stress in the contact using Eyring’s law:

\[ \eta du / dy = \tau_o \sinh(\tau / \tau_o) \]

These approximations are very limited regarding the behaviour but seem to fit very well experimental results obtained with a single contact on a pin on disc tribometer [14]. Expression of the friction force \( F_{EHD} \) in EHD mechanism becomes:

\[ F_{EHD} = T_o \left( \frac{3W_{EHD}}{2} \right)^{2/3} \left[ \frac{\pi e^{K_1 (K_2 + 6K_3)}}{24} \frac{e^{K_1^2}}{\pi \text{arcsinh}(e^{K_1} / K_o)} \right] \]

The limit of the EHD domain corresponds to the beginning of plastic deformation in a part of the contact.

Adopting a Tresca Criteria for stresses the expression of the limit \( H_{EHDlim} \) is:

\[ H_{EHDlim} \equiv 1.228U \left( \frac{G}{2E_q} \right)^{0.55} \left( \frac{R_o}{2E_q} \right)^{0.201} - 109.662 \left( \frac{R_o}{2E_q} \right)^2 \]

5.3 ElastoPlastic mechanism (EP)

Elastic and Plastic deformation coexist in the contact. The ratio between the two kinds of deformation depends on the contact conditions. We use theoretical results first proposed by Hill and reported by Johnson [15]. These results concern a spherical indenter and give a relation between the mean pressure and the radius of the contact area. If we adopt the classical dimensionless parameters for the normal load \( W_{EP} = W_{EP} / (E R^2) \) and the radius of contact \( A = a / R \), we obtain:

\[ W_{EP} = \frac{2\pi R_o}{3} A_{EP} \left[ 358 + \log \left( \frac{E}{R_o} \right) A_{EP} \right] \]

Chang [16] proposed a link between the radius of the contact area and the indentation. This link is based on the classical assumption of matter conservation during plastic deformation. Redistribution of matter is localised near the contact and conserves a rounded shape.

\[ A_{EP}^2 = 2\Delta_{EP} - 27.416 \left( \frac{R_o}{E_q} \right)^2 \]

Elastoplastic regime is assumed to be a direct contact and we use Coulomb’s law to estimate the friction force. The knowledge of the level of friction \( f \) in the limit regime allows the determination of the friction force applied on one asperity.

5.4 Full Plastic mechanism (P)

The plastic deformation mechanism corresponds to a generalised plastic deformation in the contact. Thus the mean contact pressure \( p_m \) remains constant at the value:

\[ p_m = 3R_o \]

As for elastoplastic deformation, we use a matter conservation assumption based on Chang’s relation between indentation and radius of the contact area. The use of the dimensionless parameters gives:

\[ W_{EP} = 3\pi \left( \frac{R_o}{E_q} \right) A_{EP}^2; \quad A_{EP}^2 = 2\Delta_{EP} - \Delta_{PEP} \]
Assuming the continuity with the elastoplastic domain, full plastic deformation begins at the interference:

\[ \Delta_{\text{EP}} \approx 3980 \left( \frac{R_{\text{pe}}}{E} \right)^2 \]

Such a situation, as for elastoplastic regime, corresponds to a direct contact. The friction force is considered to be proportional to the normal load.

6 THEORETICAL RESULTS

Characteristics of the contact are summarised in Table 2. As can be seen, the studied contact is between two rough deformable surfaces. As a great majority of authors [8, 17], we use the sum surface to transform the contact studied in a fictive contact between a smooth rigid surface and a rough deformable one - the so-called sum surface- which combines the roughness and the deformability of the initial surfaces.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>contact</td>
<td>Plan/ plan (10 mm² area)</td>
</tr>
<tr>
<td>Lubricant</td>
<td>( \eta = 0.0133 \text{Pas} ), ( \tau = 1.98 \times 10^7 \text{Pa} )</td>
</tr>
<tr>
<td>Load</td>
<td>ranging 40 to 400 N</td>
</tr>
<tr>
<td>Speed</td>
<td>ranging 0.2 to 5 m/s</td>
</tr>
<tr>
<td>Microgeometry</td>
<td>Face turned/ face turned or Smooth/ face turned</td>
</tr>
<tr>
<td>Materials Steel</td>
<td>Steel on Brass</td>
</tr>
<tr>
<td>Brass</td>
<td>Steel on Brass</td>
</tr>
<tr>
<td></td>
<td>( E_1 = 2 \times 10^{11} \text{Pa} ); ( \nu_1 = 0.3 )</td>
</tr>
<tr>
<td></td>
<td>( E_2 = 1 \times 10^{11} \text{Pa} ); ( \nu_2 = 0.3 )</td>
</tr>
<tr>
<td></td>
<td>( R_{\text{pe}} = 180 \text{MPa} )</td>
</tr>
</tbody>
</table>

Table 2: Parameters of the studied contact

In the following, we propose to associate elastoplastic and full plastic situations. The new family obtained first concerns asperities in direct contact with the antagonist and secondly associates all the asperities having amount of plasticity.

6.1 Proportion of asperity working with each mechanism

The number of asperities in a specific domain \( N_{\text{mech}} \) can be expressed by:

\[ N_{\text{mech}} = N_0 \int_{Z_{\text{min}}}^{Z_{\text{max}}} \int_{R_{\text{min}}}^{R_{\text{max}}} f_1(Z)^* f_2(R)^* \text{d}Z \text{d}R \]

\( R_{\text{min}} \), \( R_{\text{max}} \), \( z_{\text{min}} \), \( z_{\text{max}} \) are the limit of the considered domain.

In the following, we propose to associate elastoplastic and full plastic situations. The new family obtained first concerns asperities in direct contact with the antagonist and secondly associates all the asperities having amount of plasticity.

Analysis of the number of asperities by family is detailed in figure 7. We propose the evolution of the proportion of each family with the distance \( d \) between the mean line of the rough surface and the smooth one. The lower this parameter the greater the normal load. We examine the extreme speed 0.2 and 5 m/s.

At a given speed, the increase of the load (decrease of \( d \) parameter) promotes direct contact instead of fluid contact. The increase of speed decreases the ratio of direct contact.

6.2 Normal load transmitted by asperity

Normal load transmitted by all asperities working with each mechanism \( Q_{\text{mech}} \) express as:

\[ Q_{\text{mech}} = N_0 \int_{Z_{\text{min}}}^{Z_{\text{max}}} \int_{R_{\text{min}}}^{R_{\text{max}}} f_1(Z)^* f_2(R)^* W_{\text{mech}} \text{d}Z \text{d}R \]

Where \( R_{\text{min}} \), \( R_{\text{max}} \), \( z_{\text{min}} \), \( z_{\text{max}} \) are the boundaries of the considered mechanism and \( W_{\text{mech}} \) the respective expression for the normal load on a single asperity working with this mechanism. From this relation, normal load appears as a function of the distance between the mean line of the rough surfaces.

The normal load, plotted for each family and with the two extreme speeds is shown in Figure 8.
A large distance between the two surfaces (lowest global load) corresponds to the domain where the normal load obtained with the different mechanisms is the most equivalent. Parameters such as indentation (or normal load) or speed greatly change the hierarchy between these mechanisms.

At a low distance between the two surfaces (largest global load) major load capability acts through dry contact (EP family) and the speed parameter does not induce significant changes.

In this case parameters such as speed or normal load do not significantly change the hierarchy between the 3 families.

6.3 Global friction coefficient at the contact

The friction force for all asperities working with a same mechanism $T_{\text{mech}}$ corresponds to the following expression:

$$T_{\text{mech}} = N_0 \int_{z_{\text{min}}}^{z_{\text{max}}} \int_{R_{\text{min}}}^{R_{\text{max}}} f_1(Z) \cdot f_2(R) \cdot F_{\text{mech}} \, dZ \, dR$$

In this expression $R_{\text{min}}$, $R_{\text{max}}$, $z_{\text{min}}$, $z_{\text{max}}$ are the domain boundary of the considered mechanism and $F_{\text{mech}}$ is the friction force on a single asperity working with this mechanism.

The apparent friction coefficient of the contact is the ratio of all friction forces at the contact to the global normal load.

The representation of the evolution of this parameter with the $\eta V / p$ parameter is shown in Figure 9.

![Figure 9: Global friction coefficient evolution with the $\eta V / p$ parameter](image)

As expected, low global friction coefficient corresponds to the domain where fluid load capability is the greatest (low load and high speed). In addition, using the classical parameter $\eta V / p$, we notice that the model shows that the evolution of the friction coefficient is very similar to that of a master curve. This observation agrees very well with experimental results concerning the same contact.

6.4 Influence of the roughness

Change in roughness (Figure 10) causes modification in the master curve. A low microgeometry increases the fluid load carrying capability for the contact and decreases the global friction coefficient.

![Figure 10: Roughness influence on the global friction coefficient](image)

7 CONCLUSIONS

We have proposed a predictive model for mixed regime in lubricated contact between two parallel surfaces. This model is based on the analysis of the phenomena acting at each asperity. The main advantage of the proposed approach is to enhance the classical analysis with an isoviscous hydrodynamic situation concerning a part of the asperities and an elastic dry contact on the rest. We have proposed several situations such as piezoviscous hydrodynamic complementarity, elastohydrodynamic, elasto plasticity or full plasticity.

For a given family, the model allows us to quantify the number of asperities, the normal load and the friction forces.

Such characteristics are undoubtedly extremely useful for the analysis of the tribological behaviour of the contact.

First, the simulations prove that the newly proposed asperity families explain significant aspects of contact behaviour. Moreover, the analysis of the evolution of the friction coefficient with the parameter $\eta V / p$ accords surprisingly well with the experimental results.

8 REFERENCES

9 NOTATIONS

\[ a : \text{ Radius of contact} \]
\[ h_0 : \text{ Film thickness} \]
\[ Alt = R / 2 : \text{Mean value of the summit} \]

\[ AR : \text{Mean width of the motif's} \]
\[ A = a / R : \text{Dimensionless radius of contact} \]
\[ E_i : \text{Young modulus of surface } "i" \]
\[ f' : \text{Coefficient of friction} \]
\[ F : \text{Dimensionless friction force} \]
\[ G = \alpha E_i : \text{Material parameter} \]
\[ H_0 = h_0 / R : \text{Dimensionless film thickness} \]
\[ H_{\text{EHD} \text{lim}} : \text{Limit film thickness in EHD regime} \]
\[ H_{\text{PVR} \text{lim}} : \text{Limit film thickness in PVR regime} \]
\[ N_a : \text{Total number of asperity on the whole surface} \]
\[ p_a = Q / \pi a^2 : \text{Mean pressure} \]
\[ Q : \text{Normal load} \]
\[ R : \text{Mean height of the motif's} \]
\[ R : \text{Radius of asperity} \]
\[ R_{\text{pe}} : \text{Yield stress of the softer material} \]
\[ S_{\text{Alt}} = 0.35 \sqrt{W^2 + SW^2} : \text{Standard deviation of the distribution} \]
\[ S_{AR} : \text{Mean square of the motif's width.} \]
\[ S_{R} : \text{Root mean square of the motif's height} \]
\[ S_{W} : \text{Root mean square of peaks amplitude.} \]
\[ T : \text{Friction force} \]
\[ u_i (i = 1, 2) : \text{Speed of solid 1, 2} \]
\[ U = \eta_o \left( h_0 + u_i \right) / 2 \varepsilon R : \text{Dimensionless speed} \]
\[ V : \text{Sliding velocity} \]
\[ W : \text{Mean vertical amplitude between peak} \]
\[ W = \left( Q / ER^2 \right) : \text{Dimensionless normal load} \]
\[ E_q = 2 / \left( \left( 1 - v_1^2 \right) / E_1 + \left( 1 - v_2^2 \right) / E_2 \right) : \text{Equivalent Young modulus} \]
\[ \alpha : \text{Lubricant piezoviscosity coefficient} \]
\[ \delta : \text{Asperity deformation.} \]
\[ v_1 : \text{Poisson coefficient of surface } "i" \]
\[ \eta_o : \text{Viscosity} \]
\[ \Delta = \left( \delta / R \right) : \text{Dimensionless deformation} \]
\[ \Delta_{\text{EP}} : \text{Elasto-hydrodynamical limit} \]